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# USING FLIGHT MANUAL DATA TO DERIVE AERO-PROPULSIVE MODELS FOR PREDICTING AIRCRAFT TRAJECTORIES

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**Abstract**

The Center/TRACON Automaton System (CTAS) is a set of air traffic management tools developed by NASA in conjunction with the FAA. As part of its functionality, CTAS predicts aircraft flight trajectories using aero-propulsive models and the kinetic equations of motion for various flight conditions including climbs. Precise aero-propulsive models for all aircraft types are not yet available to NASA researchers. In an effort to improve climb trajectory prediction of jet aircraft for which CTAS does not have a precise aero-propulsive model, a technique was developed to derive an aero-propulsive model from readily available time-to-climb data found in flight manuals. A case study was performed on a Boeing 737-400, for which time-to-climb data and aero-propulsive model data were known. A new aero-propulsive model, identified by three aerodynamic and one propulsive parameter, was derived from the time-to-climb data. The results showed it was possible to derive an aero-propulsive model for an aircraft type that will allow CTAS to compute time-to-climb for a range of climb speeds that agree closely with known data. This technique was then applied to a Learjet 60, an aircraft type for which a precise aero-propulsive model is not available. A comparison of top-of-climb predictions made with a derived aero-propulsive model and actual top-of-climb from Learjet 60 radar track data reveal close agreement.

## Introduction

NASA, in conjunction with the FAA, is developing a set of decision support tools to help air-traffic managers and controllers improve flight efficiency and airspace capacity. This set of tools, known as the Center/TRACON Automation System[1] (CTAS), is designed for management of traffic within each Air Route Traffic Control Center (ARTCC) and Terminal Radar Approach Control (TRACON). At the core of CTAS is the Trajectory Synthesizer (TS), which predicts a path for each aircraft from its current position to an end point of interest, typically more than 30 minutes ahead. Predictions for aircraft climbs and descents are made by integrating kinetic equations of motion. Such equations require the use of aero-propulsive models to determine thrust and drag forces.

There are almost 400 individual FAA-designated aircraft types. Because some aircraft manufacturers are reluctant to provide aero-propulsive data, precise aero-propulsive models for all these types are not available to NASA researchers. For these aircraft types, the TS uses a representative model with comparable performance characteristics. In an effort to improve climb trajectory prediction of jet aircraft for which CTAS does not have a precise aero-propulsive model, a technique was developed to derive a model from readily available time-to-climb data found in flight manuals (also known as a pilot's manual or operations manual, hereafter referred to as "flight manual").

This paper describes a technique for deriving an aero-propulsive model from aircraft-specific climb performance data that will allow the TS to make more accurate climb trajectory predictions. The paper is organized as follows. The first section of this paper describes the methodology used for the aero-propulsive model derivation. The next section details the results of a case study in which this technique was applied to an aircraft with known aero-propulsive model data. The third section describes the results when a model was derived for an aircraft that previously did not have a precise aero-propulsive model. The paper closes with some concluding remarks.

## Methodology

An aero-propulsive model is used to compute the forces acting on an aircraft under specific flight conditions and consists of two parts. One part is used to compute the aerodynamic drag force, while the other is used to compute the propulsive thrust force. Using the kinetic equations of motion and the difference between these

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two forces, the climb performance of an aircraft can be computed for a given weight, speed schedule, and atmospheric condition. Conversely, if climb performance for a specific aircraft is known, the difference between these forces can be determined. A complementary aero-propulsive model can then be derived for that specific aircraft which models its climb performance.

The basis for this derivation technique is the time-to-climb data found in aircraft flight manuals. Time-to-climb data is usually presented for a specific climb-speed schedule, initial weight, thrust setting, and standard atmospheric conditions. A sample of the flight manual climb data published for a Learjet 60[2] is shown in Table 1.

Table 1. Sample Climb Data for a Learjet 60

| Altitude (1000 ft) | Time (min) | Distance (nm) | Fuel (lb) |
|---|---|---|---|
| 47 | 22.3 | 141.0 | 639.8 |
| 45 | 14.9 | 91.8 | 513.7 |
| 43 | 12.4 | 75.2 | 465.4 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| 5 | 0.8 | 3.6 | 49.9 |
| 3 | 0.5 | 2.1 | 30.2 |
| 1 | 0.2 | 0.7 | 10.2 |

The sample data specifies the time it takes a Learjet 60 to climb from sea level to a specified altitude with an initial weight of 18,000 lb, a climb speed schedule of 250 knots calibrated airspeed (KCAS) to Mach 0.70, and thrust set at the Maximum Climb Thrust rating. Horizontal distance flown and fuel burned during the climb are also presented. For example, a Learjet 60 can climb from sea level to 43,000 ft in 12.4 minutes, consuming 465.4 lb of fuel and travelling a horizontal distance of 75.2 nm. Aircraft weight at 43,000 ft can simply be obtained by subtracting the fuel burned from the initial aircraft weight.

Time-to-climb, t/c, can be calculated by integrating the basic kinetic equations of motion. Consider the following equation for rate of climb[3]:

$$\dot{h} = \frac{\left(\frac{T - D}{W}\right)V}{1 + AF} \qquad (1)$$

where $\dot{h}$ is rate of climb, T is thrust, D is drag, W is weight, V is true airspeed, and AF is acceleration factor. Acceleration factor is defined as follows:

$$AF = \frac{V}{g}\frac{dV}{dh}$$

where g is the acceleration due to gravity and h is altitude.

Integrating the inverse of Eq. (1) with respect to altitude results in time-to-climb.

$$t/c = \int \frac{1}{\dot{h}}\, dh = \int \frac{1 + AF}{\left(\frac{T - D}{W}\right)V}\, dh \qquad (2)$$

Equation (2) is the fundamental equation from which an aero-propulsive model is derived because it relates climb performance to aero-propulsive forces. The two unknowns in Eq. (2) are drag and thrust, the variables to be modeled. Each force is modeled separately. A total of four parameters (three aerodynamic and one propulsive) are identified in the models for each specific aircraft type. However, the form of the aerodynamic and propulsive models remains constant. All other values can be determined from the flight manual time-to-climb data.

<u>The Aerodynamic Model</u>

Development of an aerodynamic model starts with the equation for a drag polar of a cambered wing[4]:

$$C_D = C_{Dmin} + \frac{\left(C_L - C_{Lmin}\right)^2}{\pi Ae} \qquad (3)$$

where $C_D$ is total drag coefficient, $C_{Dmin}$ is minimum drag coefficient, $C_L$ is lift coefficient, $C_{Lmin}$ is minimum lift coefficient, A is aspect ratio, and e is Oswald efficiency factor. Although Eq. (3) is normally applied to a wing only, this equation was assumed to be applicable to an entire aircraft because drag polars for an entire aircraft are also parabolic in shape. It should be noted the Oswald efficiency factor is classically a wing-only value as well.

An additional term based on the Prandtl-Glauert compressibility factor is applied to Eq. (3) in order to model the variation in drag with respect to Mach number. The resulting equation is the aerodynamic model used in this study:

$$D = C_D qS$$

$$D = \left[ C_{Dmin} + \frac{(C_L - C_{Lmin})^2}{\pi Ae\sqrt{1 - M^2}} \right] qS \qquad (4)$$

where M is Mach Number, q is dynamic pressure, and S is reference wing area. The three aerodynamic model parameters to be identified are $C_{Dmin}$, $C_{Lmin}$, and e.

The Propulsive Model

The thrust used to calculate flight manual time-to-climb data is typically fixed at a specific thrust rating, normally Maximum Climb Thrust. For a specific thrust rating, corrected thrust, $T/\delta$, is a function of Mach number, altitude, and temperature. Corrected thrust is defined as the net thrust per engine, T, divided by the ambient pressure ratio, $\delta$. The thrust model for this technique was based on an existing engine model currently available to the TS. Although the magnitude of corrected thrust varies with engine type, engine behavior with respect to flight conditions is similar. Basing the propulsive model on an existing engine model allowed the relationship between corrected thrust, Mach number, altitude, and temperature to remain consistent throughout the flight envelope. Attempts to use an empirical formula to model this relationship proved to be unpredictable outside of the limited portion of the flight envelope covered by the time-to-climb data.

An engine scaling parameter, K, serves to vary the magnitude of thrust with respect to the existing engine model.

$$T/\delta = K\,(T/\delta)_{\text{existing model}} \qquad (5)$$

The thrust per engine is determined by multiplying corrected thrust, Eq. (5), by the appropriate ambient pressure ratio.

$$T = \delta\,(T/\delta) = \delta\,K(T/\delta)_{\text{existing model}} \qquad (6)$$

Model Parameter Identification

An aero-propulsive model is derived by determining a combination of thrust and drag that best models the flight manual time-to-climb data. This is accomplished by applying an iterative error minimization process, which is depicted in Fig. 1.

Time-to-climb is calculated from modeled values of thrust and drag. An error between the calculated and flight manual time-to-climb is determined. Using a minimization routine, the three drag parameters in Eq. (4) and the engine scaling parameter in Eq. (6) are adjusted such that error between the calculated and flight manual time-to-climb data is minimized. The resulting parameters describe the aero-propulsive model.

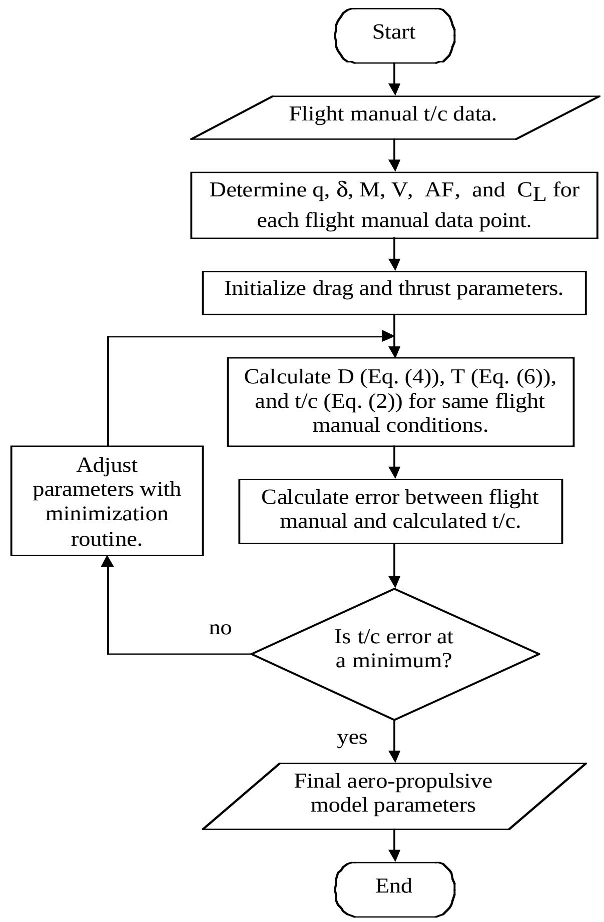


Fig. 1 Model Parameter Identification

## Case Study

Validation of the methodology was accomplished by performing a case study on an aircraft type for which time-to-climb data and aero-propulsive model data were known. An aero-propulsive model was derived for the case study aircraft. Time-to-climb predictions calculated with the derived aero-propulsive model were then compared to time-to-climb data calculated with manufacturer-supplied aircraft performance software.

A Boeing 737-400 with CFM56-3B-1 engines was chosen as the aircraft for the case study. In a cooperative research effort with Boeing, NASA researchers obtained the Boeing INFLT aircraft performance software, which can be used to calculate aircraft performance of most Boeing aircraft types.

This software was the source of Boeing 737-400 climb performance data from which the aero-propulsive model was derived and compared. Since flight manuals generally publish data for a single, recommended climb speed schedule, time-to-climb data for only one climb speed schedule was used to derive the aero-propulsive model. For the 737-400 case study, a climb speed schedule of 250 KCAS to 280 KCAS to Mach 0.74 was used. Time-to-climb data was calculated for a standard atmosphere, Maximum Climb Thrust rating, and six different initial weights from 90,000 lb to 135,000 lb.

An existing engine model serves as the basis for the derived propulsive model. Many of the aircraft types for which CTAS does not have a precise model are smaller than a 737-400 (e.g. Learjet 60). When this derivation technique is applied to those aircraft, it is likely the thrust level will be scaled down from a baseline engine model. For this reason, a Pratt and Whitney PW4056, used by a Boeing 747-400, was chosen for this case study. The PW4056 has a take-off thrust rating of about 56,000 lb per engine, while the CFM56-3B-1 engine, used by a 737-400, has a take-off thrust rating of about 20,000 lb per engine. When the PW4056 was paired with the 737-400, its thrust level had to be scaled down in order for a valid aero-propulsive model to be derived. The resulting engine scaling parameter was on the order of that expected if this technique was applied to an aircraft such as the Learjet 60.

The 737-400 Case Study Results

The parameter identification process for the aero-propulsive model derivation (Fig. 1) was applied to the case study aircraft, a 737-400 paired with the PW4056 engine model. The resulting aero-propulsive model parameters derived from the INFLT climb performance data are shown in Table 2.

Table 2. Model Parameters for the 737-400

| Parameters | Values |
|---|---|
| $C_{Dmin}$ | 0.026 |
| $C_{Lmin}$ | 0.20 |
| e | 0.68 |
| K | 0.43 |

A comparison between the time-to-climb predictions calculated with the derived aero-propulsive model and the Boeing INFLT data for the recommended climb speed schedule is shown in Fig. 2.

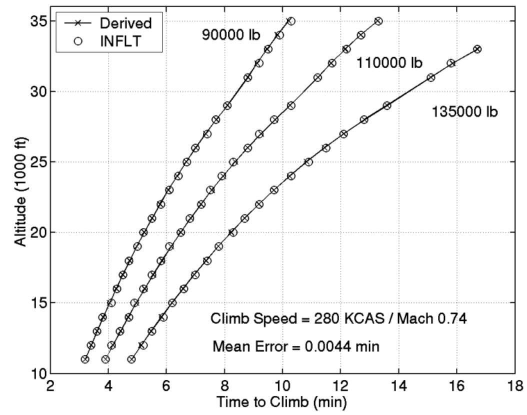


Fig. 2 Time-to-climb comparison for 737-400

It is clear the iterative derivation technique was able to find a solution that fits the source data well. Mean time-to-climb error for all weights was only 0.0044 minutes (0.3 sec). A positive error means the derived model was climbing slower than the INFLT model.

Time-to-climb data for climb speed schedules ± 20 KCAS and ± 0.02 Mach from the recommended climb speed schedule were compared in order to determine the accuracy of the model at climb speeds different from those for which it was derived. It was assumed aircraft normally climb at or relatively near the recommended climb speed schedule. Therefore, the above speed variations were expected to encompass the portion of the flight envelope at which most 737-400 aircraft climb. Comparison results for the off-condition time-to-climb predictions are shown in Fig. 3.

Mean time-to-climb error for speeds slower and faster than the recommended climb speed schedule were –0.055 minutes (-3 sec) and –0.0078 minutes (-0.5 sec), respectively. Because data for only one climb speed schedule was used in the derivation of the model, mean time-to-climb error increased for predictions made at other speeds. The accuracy of the derived aero-propulsive model would be expected to increase if data for other climb schedules were used in the derivation.

The case study results showed an aero-propulsive model can be derived from a limited amount of data. The resulting aero-propulsive model can effectively predict climb trajectories for a range of climb speeds. Derived aero-propulsive models become a viable alternative for modeling when precise aero-propulsive models are not available to CTAS.

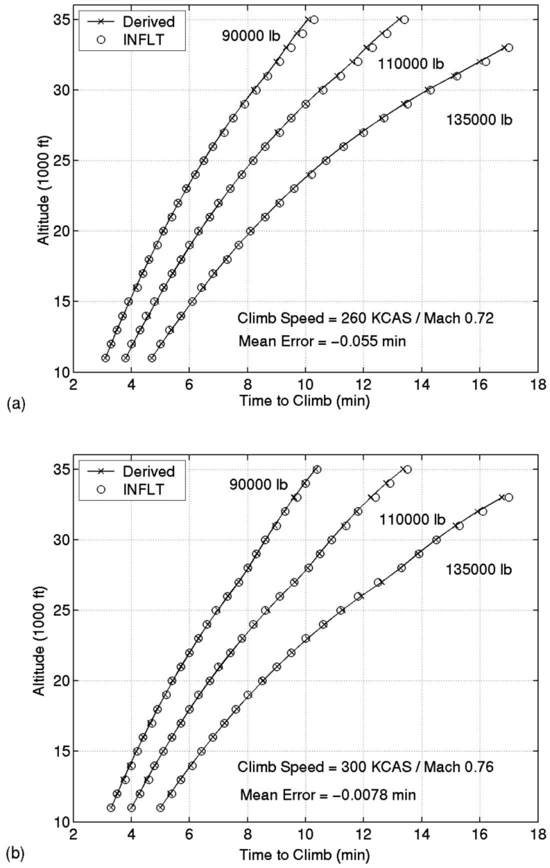


Fig. 3 Off-condition time-to-climb comparison for 737-400: (a) 20 KCAS / 0.02 Mach slower, (b) 20 KCAS / 0.02 Mach faster.

## Improving Learjet 60 Climb Predictions

The Learjet 60 is one of the aircraft types in CTAS that does not have a precise aero-propulsive model. NASA researchers identified this aircraft as a candidate for a derived aero-propulsive model. A flight manual for a Learjet 60 was obtained so that the time-to-climb data could be used for the derivation. Climb data published in this manual were for a climb speed schedule of 250 KCAS to Mach 0.70, Maximum Climb Thrust rating, and twelve different initial weights ranging from 14,000 lb to 23,500 lb. Only the standard atmosphere time-to-climb data were used for the derivation.

A CFM56-3B-1, with a thrust rating of about 20,000 lb, was selected as the base engine for the propulsive model. Currently, NASA researchers do not have business jet engine models with thrust ratings on the order of 5,000 lb. However, the case study demonstrated a base engine for the propulsive model could be scaled down significantly and still be effective.

The aero-propulsive model parameters derived from the Learjet 60 flight manual data are shown in Table 3.

Table 3. Model parameters for the Learjet 60

| Parameters | Values |
|---|---|
| $C_{Dmin}$ | 0.015 |
| $C_{Lmin}$ | 0.12 |
| e | 0.71 |
| K | 0.22 |

Time-to-climb predictions calculated with the derived model compared well with the published flight manual data as depicted in Fig. 4. Mean time-to-climb error for all data was -0.017 minutes (-1 sec).

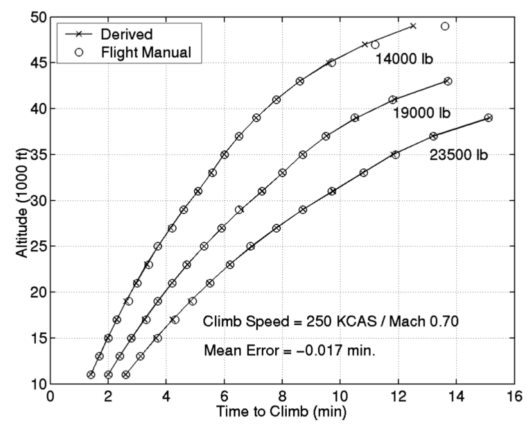


Fig. 4 Time-to-climb comparison for Learjet 60

The derived aero-propulsive model was then implemented in CTAS in order to determine any improvement over the current model. Time-to-climb predictions calculated with each model were compared to Learjet 60 radar track data recorded from the Dallas-Fort Worth ARTCC. The radar track data included altitude measurements received from the on-board mode C transponders of each aircraft.

About 48 hours of radar track data for Learjet 60 aircraft were recorded over a period of several days. Because actual radar track data were used, the climb segments of many Learjet 60 flights contained interruptions such as temporary altitudes. Only flights that reached top of climb without interruption were used for this study.

A comparison of time-to-climb predictions and radar track data for one of the seventeen Learjet 60 flights used in this study is shown in Fig. 5. Time-to-climb was measured from 13,000 feet, the altitude of the aircraft when the predictions were made.

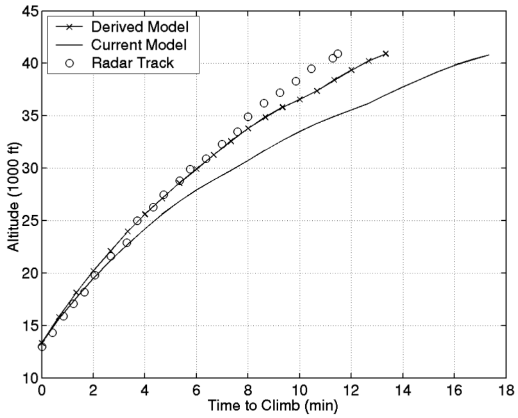


Fig. 5 Learjet 60 radar track versus predictions

The climb trajectory predicted by the derived model matched the actual radar track better than the current model. A marked improvement in the top-of-climb prediction was evident. Top-of-climb prediction error using the derived model was about two minutes versus more than six minutes with the current model. In addition to modeling error, a number of other factors such as wind prediction, temperature profile, and aircraft weight may contribute substantially to the top-of-climb prediction error. Results were similar for the other sixteen flights.

In a real time air traffic management environment, the TS predictions are recomputed every twelve seconds as the position of the aircraft changes. Figure 6 depicts the error in the top-of-climb predictions as the same aircraft climbed to its cruise altitude of 41,000 feet.

The overall error in a set of predictions was represented by a figure of merit (FoM). The FoM is a cumulative, weighted measure that increases the penalty on the prediction error as the altitude from which the prediction is made approaches the cruise altitude[5]. A lower FoM indicates a better set of predictions. For this flight, the FoM of the predictions calculated with the derived and current models was 0.89 and 2.11, respectively. Top-of-climb prediction error decreased by almost 60% when using the derived model.

For all seventeen flights, the mean FoM of the predictions calculated with the derived model was 0.95 with a standard deviation of 0.78. The predictions with the current model had a mean FoM of 2.38 with a standard deviation of 1.28. Similarly, the overall decrease in top-of-climb error was 60%. The use of a derived aero-propulsive model significantly improved top-of-climb predictions over the current Learjet 60 model.

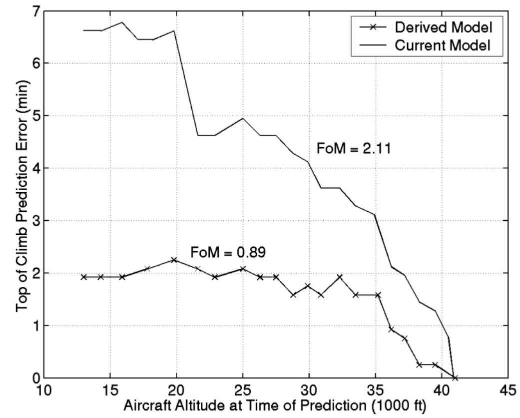


Fig. 6 Learjet 60 Top of Climb Prediction Error

## Concluding Remarks

Air traffic management tools, such as the NASA developed CTAS, promise to improve the efficiency and capacity of the National Airspace System. NASA researchers are continually improving CTAS. Towards this end, a technique for deriving aero-propulsive models was developed and evaluated.

The results of the evaluation showed aircraft-specific performance data, found in readily available flight manuals, can be used to derive an effective aero-propulsive model. For the Boeing 737-400 case study, mean errors for the time-to-climb predictions made with a derived model versus a manufacturer's model were no more than three seconds for a range of climb speeds. Such a model proved to be a viable alternative if a precise aero-propulsive model was unavailable. When a derived model was applied to the Learjet 60, top-of-climb prediction error versus actual radar track data decreased by 60% over predictions made with the current, imprecise model.

Although flight manual data was used in this study, this technique does not preclude the use of other data sources. Aero-propulsive model parameters may be identified from radar track data. However, a larger quantity of radar track data would be required as these data are of a lower quality than flight manual data. This is because radar track data introduce the influences of other factors that may not be accurately determined. Collecting a sufficient amount of usable radar track data for the Learjet 60 comparison did prove to be time consuming during this study.

Improvements to the derived aero-propulsive model can be achieved if additional aircraft-specific performance data become available. The use of a more representative base engine model for the propulsive portion of the model will contribute to prediction accuracy. Additional research into the aerodynamic model equation may also offer improvement.

Future studies will investigate the prediction accuracy of the derived model for other flight conditions, such as descents. Preliminary descent study suggests changes to the derived model may be required because the engine scaling parameter may not effectively model idle thrust and ram drag effects. Descent trajectory calculations are further complicated when flight manual descent data have cabin pressurization rate or other limitations applied. In principle, however, a derived model could improve descent trajectory predictions if such data were included in the parameter identification process. The accuracy of the derived model with respect to trajectory predictions for other flight conditions is still under investigation.

## Acknowledgements

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