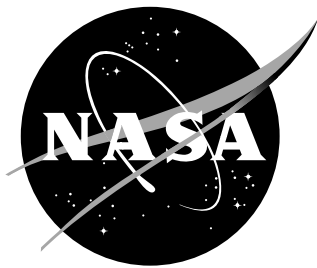NASA/TM–2014–216618

Schedule Dissimilarity and Stability Metrics for Robust Precision Air Traffic Operations

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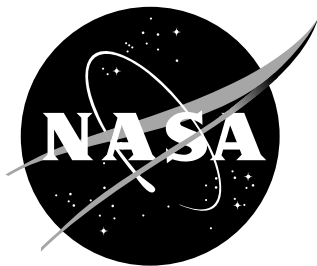- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

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Abstract

The Next Generation (NextGen) Air Transportation System and the Single European Sky ATM Research (SESAR) programs aim to overhaul and modernize the existing air transportation systems in the United States and Europe, respectively. The NextGen and SESAR concepts of operation leverage procedures based on the Global Positioning System and modern aircraft avionics to increase the predictability and reliability of air transportation. Air Traffic Operations (ATO) that depend on GPS-based procedures are referred to as Precision Air Traffic Operations (PATO). Such operations, although modeled deterministically in prior work, remain subject to a number of uncertainties present in the current system, the foremost of which is error in the forecast winds aloft. As a first step toward scheduling PATO in a stochastic environment, this paper proposes a precise definition of the stability of a given PATO schedule and a quantitative definition of the dissimilarity between two such schedules. These metrics are aimed at recognizing those schedules and changes to a schedule that may compromise the robustness of the PATO; for example, leading to a loss of aircraft separation or of required performance.

1   Introduction

NextGen and SESAR are envisioned to introduce widespread use of GPS-based procedures called Area Navigation (RNAV) and Required Navigation Performance (RNP) [1]. RNAV and RNP procedures are expected to increase the predictability of aircraft trajectories and allow for more strategic application of control strategies for keeping aircraft safely separated and expediting progress toward their destination. However, the envisioned predictability and efficiency increases depend on aircraft to follow the prescribed RNAV and RNP procedures. Such operations depending on adherence to precisely defined routes of flight are hereafter called Precision Air Traffic Operations (PATO).

The problem of PATO scheduling and its constraints are described in [2]. (Throughout the rest of this paper, the term *schedule* will, in deviation from the common use, refer to a vector-valued function of time, defined in detail in a later section.) The mechanisms to be used by air traffic control to separate aircraft conducting PATO are a subject of current research [3]. However, regardless of the specific form of decision support provided to air traffic controllers and flight crews in the conduct of PATO, abrupt, frequent or ill-timed schedule changes can disrupt other PATO functions (e.g. providing separation between aircraft). Such disruptions may compromise safety and will likely be deemed unacceptable by those responsible for the execution of the schedule (i.e., air traffic controllers and aircraft flight crews). Whatever the form of

decision support, the response of the PATO system to the operational uncertainties (which are described in [2]) will generally entail changes to the schedule. Therefore, to prevent aforementioned disruptions, these schedule changes must allow the air traffic control (whether human or automated) and the pilot sufficient time to react.

The purpose of this paper is to propose a quantitative measure of the response (manifested as a schedule change) of a PATO system to perturbations arising from the operational uncertainties (described in [2] and modeled here as stochastic perturbations to appropriate input parameters[1]). The primary contribution of this paper is a dissimilarity metric for an identically-routed schedule pair that allows for consideration of robustness in scheduling PATO. This capability is needed to prevent inherent ATO uncertainties (e.g. wind forecast errors) from causing the aforementioned disruptions. The dissimilarity metric proposed in this paper will allow for consideration of schedule robustness in two ways:

- as a cost in the objective function of a PATO schedule optimization, and

- as a measure of a given schedule's sensitivity to the inherent uncertainties.

The computational cost of the methods used to determine a PATO schedule will determine how the proposed metric is used in future ATO decision support or automation. Given sufficient time and computational capabilities, an optimal PATO schedule solution can be sought with an objective function that reflects schedule robustness and operational priorities (e.g. runway utilization and flight efficiency). Lacking sufficient computational resources for such an optimization, a given schedule can be evaluated to determine its sensitivity to inherent ATO uncertainties and if the sensitivity of a schedule exceeds a prescribed maximum sensitivity, the traffic demand (input) adjusted accordingly as a precautionary measure.

The contributions of this paper and the potential future use of the work herein are best understood with knowledge of the PATO scheduling problem and exposure to the prior work in scheduling (current) ATO and its relevance to PATO. A thorough description of the problem of scheduling for PATO is included in [2], and a brief review of the formal problem statement is provided in the following section along with a brief review of prior research into robust scheduling for PATO. The remainder of this paper is organized to first develop a definition for dissimilarity between two PATO schedules and then to provide concise numerical examples to demonstrate the potential utility of the proposed metric toward robust NextGen PATO. Lastly, because there will be situations in NextGen, where aircraft will require paths different from those prescribed

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[1]The stochasticity of an input parameter here is a modeling assumption that may be difficult to test for some parameters and false altogether for others.

in a schedule, we present an extension to the proposed metric which addresses the case of non identically-routed schedules.

2 Background

This section provides a formal statement of the PATO problem and brief discussion of previous research into robust scheduling of PATO.

2.1 Scheduling PATO: problem statement

Derivation of the PATO scheduling problem definition and the enumeration of the constraints on PATO scheduling are included in [2]. One seeks to route and navigate along the chosen routes a finite set

$$\mathcal{A} = \{1, 2, \ldots, A\}$$

of A flights in a route network, each flight $\alpha \in \mathcal{A}$ to go from its origin to its destination, both specified as an input to the problem. The route network is modeled as a *directed graph*, or *digraph* [4, section A.2], $G = (V, E)$. The vertices $v \in V$ are points in a Euclidean space of dimension 2 or 3, which models the physical airspace, and correspond to *waypoints* [2] and runways in the airspace. To each edge $e = (u, v) \in E$ corresponds a *rectifiable curve* [5, section 4.6-9] which, therefore, has a well-defined arc length. Henceforth, an edge $e \in E$ will be identified with the corresponding curve. A graph $G = (V, E)$ with this additional geometric setting will be called *an airspace graph*.

Each aircraft is modeled as a point moving along an edge of G. Thus, to each aircraft α we associate a path in the graph G. Such a path, being a concatenation of continuous and piecewise differentiable curves, is itself such a curve. Once an arc length coordinate $s^{(\alpha)}$ is introduced on this curve for aircraft α, the value of this coordinate at time t completely specifies the physical position of agent α at time t. It follows immediately that the time derivative of $s^{(\alpha)}(t)$ gives the agent's instantaneous ground speed along its path. The formal problem statement is as follows [2]:

Given

- *for each $\alpha \in \mathcal{A}$ a path (parameterized by arc length $s^{(\alpha)}$) in G,*

- *the full set of constraints (e.g. separation minima) on the $s^{(\alpha)}(t)$'s and on their time derivatives (e.g. airspeed restrictions),*

- *a control dynamical law for the evolution of the $s^{(\alpha)}(t)$'s, and*

- *an objective function of the vector-valued function $(s^{(\alpha)}(t))_{\alpha \in \mathcal{A}}$ of time,*

find a control strategy [6] that results in a $(s^{(\alpha)}(t))_{\alpha \in \mathcal{A}}$ *which is* feasible *[4, 6] (i.e., constraint-compliant) and, if needed, minimizes the objective function.*

Solutions to this problem have been proposed in [7] and [8] and extended to include the case of multiple possible aircraft routings in [9].

2.2 Prior ATO schedule robustness research

Because PATO is a relatively new concept, with prior ATO being subject to ad hoc maneuvering of aircraft as instructed by Air Traffic Control (ATC), only a small body of research exists regarding robust scheduling of aircraft on a priori defined paths. Research into robustness of ATO is mostly limited to the fields of airline crew and equipment scheduling, and Air Traffic Flow Management (ATFM). Chandran and Balakrishnan [10] investigated the tradeoff between arrival runway utilization and probability of a feasible solution relative to the First Come First Served (FCFS) schedule of operations. These authors demonstrated that, for a given scheduling algorithm, the relative probability of a feasible solution decreased as the scheduled runway utilization increased. The model of uncertainty in [10] was intentionally simplistic and only a handful of the problem constraints identified by [2] were considered.

3 Schedule stability and PATO robustness

A quantitative characterization of robustness for PATO is desired for two purposes, which are long-term research goals: to gauge the sensitivity of a proposed schedule to known stochastic perturbations in the inputs, and to allow for schedule optimization that includes robustness considerations. As stated in section 1, for PATO to be robust (i.e. to perform without failure and as intended across a wide range of conditions), the schedule changes made in response to operational uncertainties must *(i)* be free of effects that can disrupt other PATO functions, and *(ii)* allow the ATC and the pilots sufficient time to execute the newly provided schedule. We propose to approach this by characterizing such schedule changes and execution urgency quantitatively. The rest of this section is a development of the mathematical machinery for such a characterization.

3.1 Parameter-dependent schedules

As was noted in section 2.1, a *schedule* is a double datum, prescribed for a given set of aircraft to travel in a given route network, of the form

$$\sigma = (\text{routing, collective control strategy}), \tag{1}$$

where the *routing* refers to a function mapping each aircraft $\alpha \in \mathcal{A}$ in the given operation to a path in the route network, and the *collective control strategy* is a vector

$$\mathbf{u}(t) = \left(u^{(\alpha)}(t) \right)_{\alpha \in \mathcal{A}}$$

that prescribes the motion for all aircraft simultaneously, with the $u^{(\alpha)}$'s being the control variables in the *state equations* [5, section 11.8-1(a)] of this motion. If inertia is neglected in the model provided by the state equations, then the $u^{(\alpha)}$'s are typically the speeds; otherwise, if inertia is included, the $u^{(\alpha)}$'s are typically the accelerations, while the speeds and the arc length coordinates of the aircraft are the state variables. The theory developed in this paper is applicable to both cases, and in both cases rests on the following assumption.

Assumption 3.1. *The speeds appearing, whether as state variables or as control variables, in the state equations and in the rest of the model are the* true air speeds.

One consequence of this assumption is that one cannot, without knowing the wind field, calculate the ground speeds and flight times of the aircraft.

Both constituent components of a schedule (1), the routing and the control strategy, are functions of a parameter that takes values in some parameter space $\mathcal{P}$ (e.g., a suitable class of wind vector fields on the given airspace), which is a subset of a vector space with some *norm* [5, section 14.2-5], denoted $|| \cdot ||$. If a schedule σ corresponds to a parameter value $p \in \mathcal{P}$, this correspondence will sometimes be made explicit using the notation

$$\sigma = \sigma(p)$$

As previously stated, schedule changes must not disrupt other PATO functions. Intuitively, we will consider a schedule *stable at a given parameter value p* (or, briefly, *stable at p*) if by small perturbations to p one cannot with high probability cause "large changes in the schedule (i.e. schedule changes that disrupt other PATO functions). A mathematical model of this intuition requires, in particular, a function that measures the dissimilarity between a pair of schedules. A general class of such functions is constructed below, in section 3.2, and has the behavior of what is called in mathematics a *metric* or a *distance function* [5, definition 12-5.2].

3.2 Definition of dissimilarity for identically routed schedule pairs

In this section, we define quantitatively the *dissimilarity* between two solutions, σ_1 and σ_2, to the PATO scheduling problem [2]. The following definition will be instrumental to this discussion.

Definition 3.1. (a) *Two schedules that have the same route network, aircraft set $\mathcal{A}$, and separation and speed constraints, and differ only in the parameter value, will be called* a schedule pair. **(b)** *A schedule pair with the same routing is said to be* identically routed.

In this paper, we consider those schedule pairs that are identically routed. A generalization to the case when a perturbation to the parameter changes the routing, discussed briefly in Section 5 below, is a topic for future research.

The intuition for the definition of dissimilarity is as follows. An identically routed schedule pair will be thought the more dissimilar, the more "substantial" the operational change from one to the other. A change in the speed advisory $v^{(\alpha)}$ for an individual aircraft α is considered the more "substantial"[2] the greater the volume of instructions is to be issued by the ATC and executed by the pilot. Thus, at each point s along the aircraft's path, the qualitative form of the "instantaneous dissimiliarity" between the two speed advisories $v^{(\sigma_1;\alpha)}(s), v^{(\sigma_2;\alpha)}(s)$ furnished for α by the two respective schedules σ_1, σ_2 has the qualitative form

$$\left(\text{ the absolute difference between } v^{(\sigma_1;\alpha)}(s) \text{ and } v^{(\sigma_2;\alpha)}(s) \right). \qquad (2)$$

Once a mathematical definition for (2) is stated, the *individual, for aircraft α, dissimilarity between σ_1 and σ_2* can be defined as the integral of (2) with respect to the arc length coordinate of the flight path of α, taken along the entire path. The dissimilarity $d(\sigma_1, \sigma_2)$ (total, for the entire operation) between schedules σ_1 and σ_2 will be defined as the summation over all α of the these integrals.

The quantity in (2) can be defined as a suitable continuous and non-decreasing function of the absolute difference

$$|v^{(\sigma_1;\alpha)}(s) - v^{(\sigma_2;\alpha)}(s)|.$$

To guarantee certain smoothness, this function will be chosen as one that squares its argument. Finally, the arc length coordinate along the path of aircraft α varies from a value denoted $s^{(ENT;\alpha)}$ (entrance) to a value denoted a value denoted $s^{(EXIT;\alpha)}$ (exit).

The resulting definition of the *dissimilarity of an identically routed schedule pair σ_1, σ_2* is

$$d(\sigma_1, \sigma_2) = \sqrt{\sum_\alpha \left\{ \int_{s^{(ENT;\alpha)}}^{s^{(EXIT;\alpha)}} \left[v^{(\sigma_1;\alpha)}(s) - v^{(\sigma_2;\alpha)}(s) \right]^2 \mathrm{d}s \right\}} \qquad (3)$$

It can be proved that, on a class of schedules where each two form a schedule pair (see definition 3.1, above) and are identically routed, the

---

[2]I.e., requiring a more substantial or labor-intensive revision of the entire operation in question.

dissimilarity function $d(\cdot, \cdot)$ is a *metric* in the sense of [5, definition 12-5.2][3], and will be used in the definition of schedule stability (see Section 3.4.1).

3.3 Numerical examples of pairwise dissimilarities in a set of schedules

3.3.1 Two schedules for two aircraft whose rectilinear paths cross perpendicularly

Setting In the route network shown in Figure 1.A, consider two aircraft, $\alpha = 1$ and $\alpha = 2$, flying the respective routes (WP0.1, WP2, WP1.1), due East, and (WP0.2, WP2, WP1.2), due north:

$$\left. \begin{array}{cl} \alpha & \textbf{route} \\ \hline 1 & \text{(WP0.1, WP2, WP1.1)} \\ 2 & \text{(WP0.2, WP2, WP1.2)} \end{array} \right\} \tag{4}$$
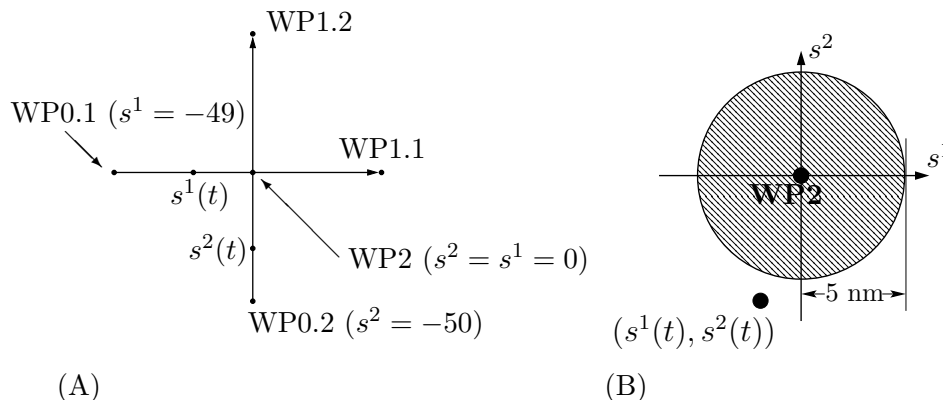

Figure 1. (A) A physical airspace with two rectilinear paths, crossing perpendicularly and each traversed by an aircraft. Each point on a path corresponds to a possible position of *one* aircraft. (B) The set of all separation-losing states in the $s^1 s^2$-*coordinate* space. Each point, (s^1, s^2), describes the positions of *both aircraft simultaneously*. The shaded interior of the circle depicts the set of all states in which the requirement for a minimal separation of 5 nm is violated.

The routes are rectilinear and mutually perpendicular segments, each parameterized by the respective arc length coordinate $s^{(\alpha)}$, $\alpha \in \{1, 2\}$,

---

[3]In other mathematical literature, the term used for the concept defined in [5, definition 12-5.2] is *pseudometric*, while a metric is defined by adding to the latter definition the requirement that if d is zero for a pair of arguments, then the arguments coincide; see, for example, Reference [11, section 1.21a].

as shown in Figure 1.A. Assume the required separation for the two aircraft, regardless of relative position, is 5 nm.

The coordinate space for this air traffic model is the set of all pairs (s^1, s^2) with $s^1 \geq -49$, $s^2 \geq -50$, with the crossing point, WP2, at the origin: $s^1 = s^2 = 0$. The set of all coordinate pairs (s^1, s^2) that have the two aircraft within 5 nm of each other, i.e. that lose separation, is shown in Figure 1.B) as the shaded interior of a circle centered at the origin and with radius 5; for detailed derivations, see [7, 9].

Suppose that the aircraft are initially at WP0.1 ($s_0^1 = -49$) nm and WP0.2 ($s_0^2 = -50$) nm, respectively, i.e. are in the initial state

$$\mathbf{s}_0 = \left(s_0^1, s_0^2 \right) = (-49, -50)$$

in the $s^1 s^2$-coordinate space, and impose the following *feasible speed range* constraint:

$$\left. \begin{array}{l} \text{Each aircraft has minimal feasible airspeed } V^{\min} = 420 \text{ kts and} \\ \text{maximal feasible airspeed } V^{\max} = 475 \text{ kts.} \end{array} \right\} \tag{5}$$

Zero wind conditions, schedule σ_1: In the absence of wind, the airspeeds coincide with the ground speeds, so constraint (5) implies that all states reachable from $\mathbf{s}_0$ lie in the cone with vertex $\mathbf{s}_0$ and bounded by the lines

$$s^2 - s_0^2 = \frac{V^{\max}}{V^{\min}} \left(s^1 - s_0^1 \right) \quad \text{and} \quad s^2 - s_0^2 = \frac{V^{\min}}{V^{\max}} \left(s^1 - s_0^1 \right)$$

This cone is schematically shown, shaded, in Figure 2.A. The Figure illustrates the following fact, which can be verified by computation: if the two aircraft are to maintain separation at all times while passing WP2, then $\alpha = 1$ must travel at maximal speed, $V^{\max}$, while $\alpha = 2$ must travel at minimal speed $V^{\min}$. This assignment of speeds, together with routing (4), is a *schedule*, which we will denote in this section by σ_1.

A constant positive northwest wind, schedule σ_2: Suppose now the presence of a wind field, constantly 12 kts due northwest. Since an aircraft will be kept, by its navigational system, on the flight path, only those components of the wind tangential to the path will affect an aircraft's movement. Consequently, the resulting ranges of feasible speeds, in reference to ground, for the two aircraft are then given by

$$V_{gs}^{\min;1} = V^{\min} - 12 \sin(\pi/4), \quad V_{gs}^{\max;1} = V^{\max} - 12 \sin(\pi/4),$$

$$V_{gs}^{\min;2} = V^{\min} + 12 \sin(\pi/4), \quad V_{gs}^{\max;2} = V^{\max} + 12 \sin(\pi/4)$$
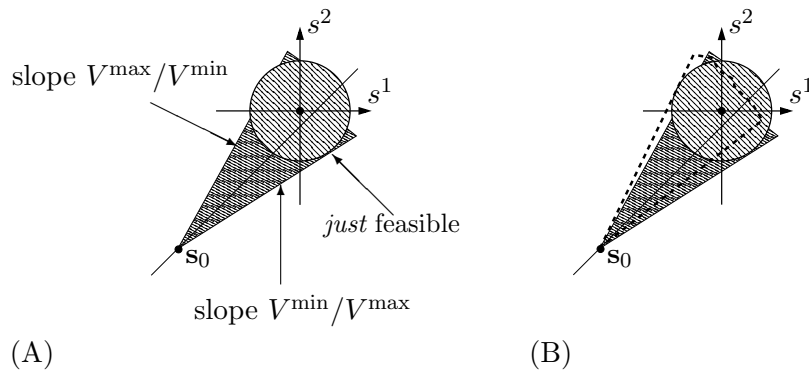
Figure 2. A schematic drawing, not to scale. In both panels, the solid line through $\mathbf{s}_0$ is the diagonal $s^1 - s_0^1 = s^2 - s_0^2$. (A) The cone (shaded) of all states reachable from $\mathbf{s}_0$ in the absence of wind is symmetric with respect to the diagonal. The only speed advisory that avoids a loss of separation is to have aircraft 1 and 2 to go constantly at speeds $V^{\max}$ and $V^{\min}$, respectively (the line with slope $V^{\min}/V^{\max}$ through $\mathbf{s}_0$). (B) The cone (dashed boundary) of all states reachable from $\mathbf{s}_0$ in the presence of a constant 12 kts wind northwest. Due to the wind's effect on each aircraft's groundspeed (reduced for aircraft 1 and increased for aircraft 2), the cone lacks diagonal symmetry. The only speed advisory that avoids a loss of separation is to have aircraft 1 and 2 progress constantly at ground speeds $V_{gs}^{\min;1}$ and $V_{gs}^{\max;2}$, respectively (ground-referenced values listed in (6)).

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which have the following approximate values:

$$
\left.\begin{array}{ccc}
\alpha & V^{(\min;\alpha)} & V^{(\max;\alpha)} \\
\hline
1 & 411.52 & 466.52 \\
2 & 428.49 & 483.49
\end{array}\right\}
\tag{6}
$$

As is illustrated in Figure 2.B (and can be verified numerically), the only speed advisory that avoids a loss of separation is to have aircraft 1 and 2 to go constantly at ground speeds $V^{\min;1}$ and $V^{\max;2}$, respectively (values listed in (6)). This schedule will be denoted by σ_2. In terms of airspeeds, σ_2 prescribes that aircraft 1 and 2 go at speeds $V^{\min}$ and $V^{\max}$, respectively.

Dissimilarity between σ_1 and σ_2   The dissimilarity (3) between the two schedules is, therefore,

$$
\sqrt{\int_{-49}^{0} [V^{\max} - V^{\min}]^2 \, ds^1 + \int_{-50}^{0} [V^{\min} - V^{\max}]^2 \, ds^2} \doteq 547.24
\tag{7}
$$

Figure 2.A shows that any increase, however slight, in the slopes of the lines that bound the shaded cone will make the speed advisory in σ_1 impossible to execute without losing separation. Consequently, the following observation (used below) holds for this example:

Remark 3.1. *With a constant wind northwest, of a magnitude strictly between 0 kts and 12 kts (see Figure 2B), schedule σ_1 is infeasible.*

3.3.2   Three schedules for two aircraft on a simple route network

This section provides a simplified PATO scheduling problem, three feasible solutions to the simplified problem, and computes the dissimilarity (Eq. (3)) to compare the sample solutions to one another. Figure 3 depicts a simple route network with two aircraft. The aircraft, 1 and 2, are 100 nm and 95 nm from waypoint WP2 along their respective flight routes, namely (WP0.1, WP1, WP2) and (WP0.2, WP1, WP2), and have no alternative flight routes (i.e., the problem is *fully routed* [7]). The flight routes merge at WP1, 50 nautical miles due north of WP2, as shown in Figure 3. Table 1 presents three feasible solutions to the problem stated in Section 2.1 subject only to constraints on the feasible speed range and the required horizontal separation between aircraft:

1. $400 \text{ kts} \leq v^{(\alpha)} \leq 500 \text{ kts}$

2. $(x^1 - x^2)^2 + (y^1 - y^2)^2 \geq r^2$, where $x^{(\alpha)}, y^{(\alpha)}$ are coordinates in the same horizontal plane as the route network and r is the required horizontal separation distance.
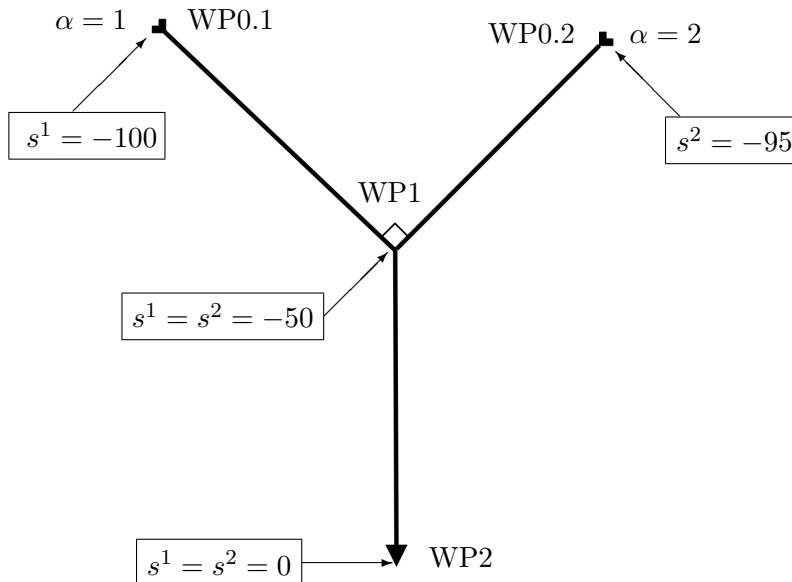
Figure 3. A simple route network with two aircraft

The values of the pairwise dissimilarities (3) for the three sample schedules are approximately:

$$d(\sigma_1, \sigma_2) \approx 5.91 \tag{8}$$

$$d(\sigma_2, \sigma_3) \approx 3.58 \tag{9}$$

$$d(\sigma_1, \sigma_3) \approx 7.16 \tag{10}$$

The dissimilarities (8), (9), (10) imply that the schedule pair (σ_1, σ_3) is more dissimilar than is the pair (σ_2, σ_3). These results match the following intuition. Schedules σ_2 and σ_3 differ only by 25 kts along the entire flight route of each aircraft (see Figures 4 and 5). Schedules σ_1 and σ_3 not only exhibit greater differences in speeds, but also prescribe different orders of aircraft arrival at WP2 (see Figure 3). Changes in aircraft arrival order indicate a significantly different schedule to the air traffic control and to pilots, as separation standards may differ and pilot visual acquisition of leading aircraft may be required. While aircraft order is not considered in schedule dissimilarity (3), reordering in congested terminal airspace would typically require large speed changes (or rerouting).

solution	aircraft	duration(s)	speed(kts)	path interval (nm)
σ_1	α_1	720	500	$[-100, 0]$
	α_2	360	400	$[-95, -55]$
		396	500	$[-55, 0]$
σ_2	α_1	758	475	$[-100, 0]$
	α_2	720	475	$[-95, 0]$
σ_3	α_1	800	450	$[-100, 0]$
	α_2	760	450	$[-95, 0]$

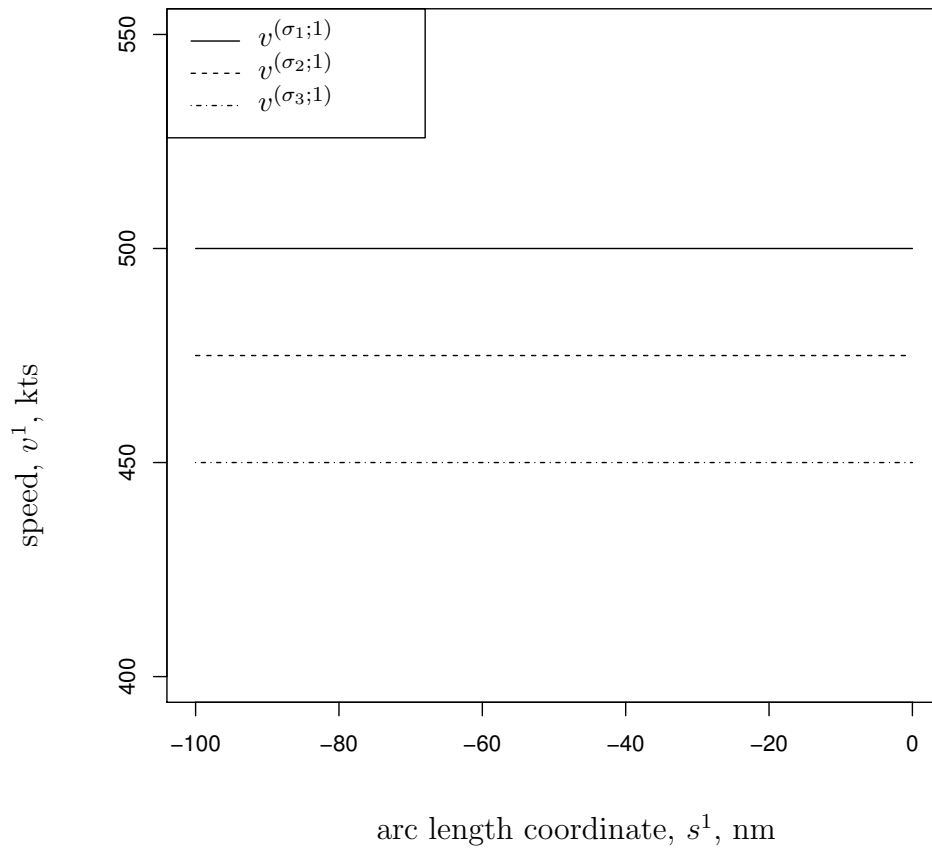Table 1. Sample solutions to example PATO scheduling problem

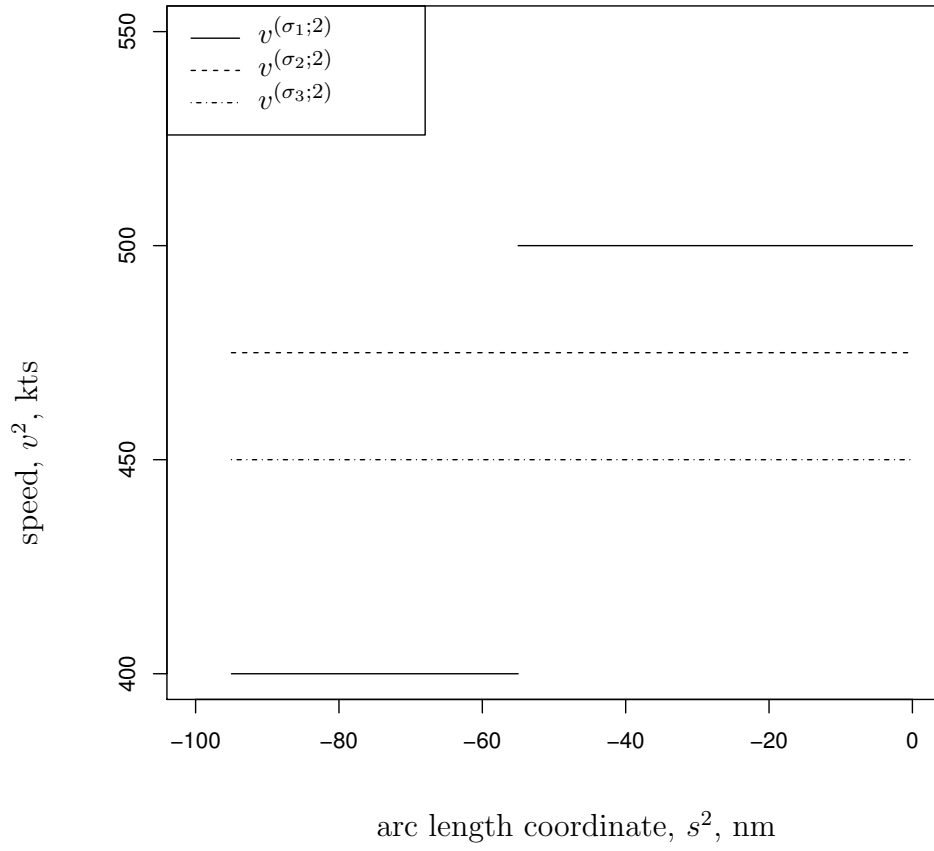Figure 4. Aircraft 1 speed advisory under each schedule.

Figure 5. Aircraft 2 speed advisory under each schedule.

3.4 Schedule stability and robustness

3.4.1 Schedule stability to stochastic perturbations

The definition (3) of dissimilarity of an identically routed schedule pair has the required capability to distinguish two schedules that, in operational intuition, substantially differ in the speed profiles. Dissimilarity will be used in this section to define stability. As indicated above, intuitively, a schedule will be thought *stable at a given parameter value* $p \in \mathcal{P}$ if by "small perturbations" to p one cannot with high probability cause large changes (in the sense of the dissimilarity (3)) in the schedule. Since this interpretation of stability is a weakened version of the notion of *continuity* [5, section 4.4-6] of a function at a point, we first recall the classical definition of continuity: a function $f(x)$ of a real argument and taking real values is said to be *continuous at a point* $x = p$ if for every positive $\epsilon > 0$ one can find a positive $\delta_\epsilon > 0$ (the subscript indicates that the choice of δ generally depends on the value of ϵ) such that

$$|f(p + \xi) - f(p)| < \epsilon \quad \text{for all } \xi \text{ satisfying } |\xi| < \delta_\epsilon. \tag{11}$$

The intuition here is that by keeping the perturbations ξ to p "small enough" (i.e., less than δ_ϵ in magnitude), one guarantees that the perturbation will not cause the value of f to move farther away from $f(p)$ than distance ϵ. This intuition connects to ATO by thinking of f as a schedule: one prefers such schedules that if a perturbation to the parameter values is "small enough," then the resulting schedule is "not too dissimilar" from the original one.

   The function f as defined above, however, does not model a schedule and its parameters p successfully: neither a schedule nor a parameter is a scalar. To address this issue, generalize by letting f take arguments from a *normed vector space* [5, section 14-2.5] with norm $||\cdot||$ and take values in a *metric space* [5, section 12.5-2] with metric $\mathbf{d}(\cdot, \cdot)$, one can define continuity by generalizing condition (11) to

$$\mathbf{d}\,(\,f(p + \xi), f(p)\,) < \epsilon \quad \text{for all } \xi \text{ satisfying } ||\xi|| < \delta_\epsilon. \tag{12}$$

Condition (12) of continuity would suit our goals completely if the values of the perturbations ξ to the parameter value vector p were deterministic. Such determinism, however, is absent from ATO: to attain satisfactory fidelity, one must make concessions. We make two, *i)* to model ξ as a random variable that takes values in the same parameter space $\mathcal{P}$ from which p comes, and *ii)* to relax the first inequality in (12) by allowing it to hold only with some confidence $(1 - r)$ (in the sense of the probability measure $\mathbf{Pr}$ that governs ξ). With these concessions, one obtains a generalization of (12) to

$$\mathbf{Pr}\Big[\mathbf{d}\,(\,f(p + \xi), f(p)\,) < \epsilon\Big] \geq 1 - r \quad \text{for all } \xi \text{ satisfying } ||\xi|| < \delta_\epsilon. \tag{13}$$

Finally, since ξ is a random variable, inequality $||\xi|| < \delta_\epsilon$ can be interpreted as an event in the probability space underlying ξ, so (13) can be written as the conditional probability

$$\mathbf{Pr}\Big[\mathbf{d}\left(\,f(p+\xi,f(p)\,)<\epsilon\,\Big|\,||\xi||<\delta_\epsilon\right]\geq 1-r \qquad (14)$$

To tie the model to ATO, let $\mathbf{d}$ be the dissimilarity metric d defined in (3), and let f be the schedule σ. The result is the following definition:

Definition 3.2. *A schedule $\sigma(p)$ is* stable *at parameter value p if for every positive epsilon ϵ and for every positive $r < 1$ there exists a positive δ_ϵ such that*

$$\mathbf{Pr}\Big[d\left(\,\sigma(p+\xi,\sigma(p)\,)<\epsilon\,\Big|\,||\xi||<\delta_\epsilon\right]\geq 1-r \qquad (15)$$

3.4.2 Robustness of PATO

PATO can be seen to consist of two major components: compilation of detailed schedules, and execution of those schedules. Thus, disruptions to a PATO can be of two major types: perturbations to the input parameters of a schedule, and perturbations to the execution of a schedule.

Perturbations of the first type, as well as the stability of a schedule to such perturbations, are defined in the preceding section. Those aspects of PATO robustness corresponding to the perturbations of the second type are less amenable to a mathematical characterization and depend on the functional architecture of the PATO system. The definition of a schedule in section 2.1 allows one to view a schedule as a control strategy for a given control system. This view, in turn, helps pinpoint a clear meaning of "an execution" of a schedule: an engineering system executes a control strategy by receiving that strategy as an input signal and by responding to this signal. In this, control-theoretic, framework, perturbations to an execution can be viewed as perturbations (whether stochastic or not) to the right-hand side of the *state equations* [5, section 11-8.1(a)], of the general form

$$\frac{\mathrm{d}}{\mathrm{d}t}(\text{state of the system}) = \begin{pmatrix} \text{a function of:} \\ \text{the state, the control, and,} \\ \text{possibly, other parameters} \end{pmatrix}, \qquad (16)$$

of the control system in question.

It follows that, to obtain a precise and general definition of PATO robustness, one must first assume some mapping that associates to each given schedule some *mechanism of schedule execution* (MSE), such as a plan of ATC clearance issuances or an automated system for reading the control strategy and conveying it in some form to the parties or devices responsible for the control of the aircraft. Assuming such a mapping, one can make the determination whether the MSE corresponding to a given schedule is "operationally feasible," i.e. is capable of being carried out

without violating PATO constraints (e.g., the required separation and feasible speed ranges; see [2] for a detailed list of such constraints). With the MSE and the capability to determine operational feasibility, one can say that a given PATO is *robust to the set Q of perturbations to the execution* if the MSE[4] corresponding to a schedule σ remains operationally feasible upon undergoing any perturbation $q \in Q$. Deeper research into the characterization of MSEs and of schedule-to-MSE mappings is a topic for future research and lies beyond the scope of this paper.

4 Numerical examples

4.1 An unstable schedule

In the example of section 3.3.1, let $\mathcal{P}$ be the set of all wind fields that are collinear (but not necessarily co-directional) with northwest and have a magnitude constant both in time and in space. Thus, a wind field is completely determined by a scalar, $p \in \mathcal{P}$, positive if the wind blows northwest. The parameter space $\mathcal{P}$ is, therefore, the real number line. The schedule σ_1 given in that example corresponds to the parameter value $p = 0$, i.e. $\sigma_1 = \sigma(0)$. The perturbations ξ to a given wind field p come, again, from $\mathcal{P}$ and follow some probability distribution, whose specific form will not be needed in this example. The norm of a perturbation $\xi \in \mathcal{P}$ will be simply the absolute value $|\xi|$. The perturbation ξ will be taken positive if the wind is directed northwest, and negative otherwise.

To show that σ_1 is not stable in the sense of definition 3.2, pick a positive ϵ smaller than the right-hand side of computation (7):

$$0 < \epsilon < 547.24$$

It follows from Remark 3.1 and computation (7) that for every positive ξ, no matter how small, the resulting schedule $\sigma(\xi)$ will be dissimilar to $\sigma(0)$ at least by amount 547.24. Consequently,

$$R_{\sigma(p);\delta,\epsilon} = 0,$$

i.e. condition (15) fails for every $\delta > 0$.

4.2 A schedule stable in the presence of a wind perturbation

Taking the route network, aircraft set, routing, and feasible airspeed ranges as in the example of section 3.3.1, we now modify the initial state

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[4]The wording "the MSE" in the latter sentence assumes that each schedule is mapped to exactly one MSE. This need not be the case: a schedule may be mapped to a set M of MSEs. In this case, the latter definition of robustness can be modified to say that *at least one MSE in M* remains operationally feasible upon undergoing any perturbation $q \in Q$.
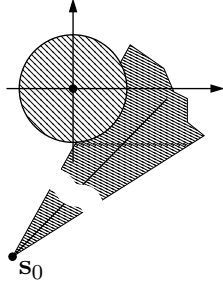
Figure 6. An example of a stable schedule $\sigma(0)$ (solid diagonal ray from $\mathbf{s}_0$) for zero wind. If a perturbation which is a constant wind northwest or southeast is sufficiently small, then the new schedule is sufficiently close to the one shown.

$\mathbf{s}_0$ so that the shaded disc, corresponding to states with lost separation, lies above the diagonal through $\mathbf{s}_0$. This situation, with the cone of states reachable from $\mathbf{s}_0$ in the absence of winds, is shown in Figure 6. Here we restrict attention to those schedules that assign to each aircraft α a constant airspeed $v^{(\alpha)}$ (allowing, however, the two aircraft to go at different speeds). On this set of schedules, we introduce the objective function (to be maximized) defined in terms of ground speeds:

$$v_{gs}^1 + v_{gs}^2$$

Thus, the objective is to move the aircraft along their routes as quickly as possible, and at speeds whose values are as close as possible. The objective function attains a maximum in the corner $(V^{\max}, V^{\max})$ of the square $V^{\min} \leq v^{(\alpha)} \leq V^{\max}$, $\alpha = 1, 2$, so an optimal schedule $\sigma(0)$ for zero wind is

$$v_{gs}^1 = v_{gs}^2 = V^{\max}$$

With the parameter space $\mathcal{P}$ as in section 4.1 (i.e., uniform and constant wind fields collinear with, or opposite to, the northwest direction), each perturbation $\xi \in \mathcal{P}$ results in the unique optimal schedule

$$\sigma(0 + \xi): \quad v_{gs}^1 = V^{\max} - \xi \sin(\pi/4), \quad v_{gs}^2 = V^{\max} + \xi \sin(\pi/4),$$

which is executable for sufficiently small $|\xi|$. The dissimilarity between $\sigma(0)$ and a generic $\sigma(0 + \xi)$, however, is computed in terms of the airspeeds, and in this case comes out to

$$d(\sigma(0), \sigma(0+\xi)) = \sqrt{\int_{-49}^{0} [V^{\max} - V^{\max}]^2 \, ds^1 + \int_{-50}^{0} [V^{\max} - V^{\max}]^2 \, ds^2} = 0$$

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It follows that $R_{\sigma(p);\delta,\epsilon} = 1$ for all sufficiently small $\delta > 0$, and consequently schedule $\sigma(0)$ is stable (at parameter value $p = 0$).

4.3 A schedule stable in the presence of a control execution timing perturbation

Consider the same route network, aircraft set $\mathcal{A} = \{1, 2\}$, and routing as in section 4.2. Stability of a schedule will be shown here qualitatively, and there will be no need for specific values of $V^{\min}, V^{\max}$, and $\mathbf{s}_0$. The only assumption here is that the initial state $\mathbf{s}_0$, as shown in Figure 6, is such that having the two aircraft go at equal ground speeds will lead to a loss of separation. Let $s^{(EXIT;\alpha)}$ be the arc length coordinate at which the flight path of aircraft α ends. In Figure 7, the shaded disc represents
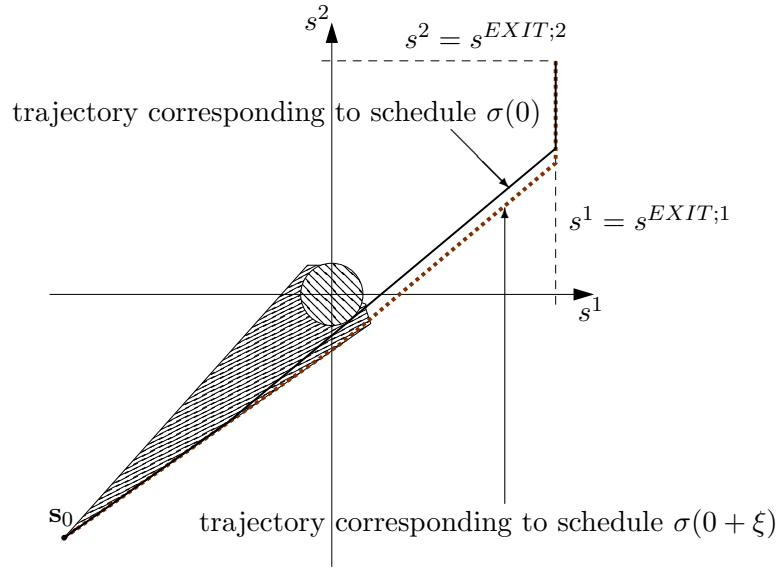

Figure 7. An example of a stable schedule $\sigma(0)$ (solid diagonal ray from $\mathbf{s}_0$) for zero wind. If a perturbation which is a constant wind northwest or southeast is sufficiently small, then the new schedule is sufficiently close to the one shown.

the set of all states in which separation is lost, and the intersection of the two dashed lines

$$s^1 = s^{EXIT;1}, \quad s^2 = s^{EXIT;2} \tag{17}$$

is the state in which both aircraft have traversed their paths completely.

The ground speed advisories (v_{gs}^1, v_{gs}^2) admitted for use here are the ones that *i)* are continuous and piecewise linear, and *ii)* reach the union of the lines (17) at a 45-degree angle to both lines (this means that at the

instant when a first aircraft reaches the end of its path, the two aircraft are going at the same ground speed). Recall that each speed advisory, $(v_{gs}^1(t), v_{gs}^2(t))$, gives rise to a trajectory $(s^1(t), s^2(t))$ in the arc length coordinate space; this trajectory is the solution to the ODE system

$$\frac{ds^{(\alpha)}}{dt} = v_{gs}^{(\alpha)}, \alpha \in \mathcal{A}$$

with the initial condition

$$(s^1(0), s^2(0)) = \mathbf{s}_0$$

The objective function used here is

$$\frac{\text{(percentage of the entire trajectory length parallel to the diagonal)}}{\text{(\#cusps in a speed advisory)}} \tag{18}$$

The parameter here is *control latency* (see [2, appendix A.2]). To capture this, the parameter space is taken to be the nonnegative ray of the real number line: $\mathcal{P} = \{p : p \geq 0\}$, and the norm $||\cdot||$ will be the absolute value $|\cdot|$. The parameter values $p \in \mathcal{P}$ are to be measured in the units of time. Assume that the perturbations $\xi \in \mathcal{P}$ obey some probability density function $f(\xi)$ which is continuous on all of $\mathcal{P}$.

With control latency $p = 0$, the schedule $\sigma(0)$ gives a speed advisory $\left(v_{gs}^{\sigma(0);1}(s^1), v_{gs}^{\sigma(0);2}(s^2)\right)$ that maximizes (18) and leads to the trajectory shown in Figure 7 as a solid polygonal curve. If the initial instantaneous ground speeds of the two aircraft are as with $p = 0$, a control latency ξ will delay execution of the same advisory, resulting in a schedule $\sigma(0+\xi)$ with a new speed advisory, $\left(v_{gs}^{\sigma(0+\xi);1}(s^1), v_{gs}^{\sigma(0+\xi);2}(s^2)\right)$. The trajectory (shown in Figure 7 as a dotted polygonal curve) corresponding to the latter advisory reaches its diagonal segment later than does the trajectory for $\sigma(0)$.

Simple geometric considerations show that

$$\lim_{\xi \to 0^+} d(\sigma(0), \sigma(0 + \xi)) = 0$$

Thus, for a given $\epsilon > 0$, the continuity of $f(\xi)$ implies

$$\lim_{\xi \to 0^+} \Pr\left[d(\sigma(p), \sigma(p + \xi)) < \epsilon \mid |\xi| < \delta\right] = 1,$$

which, in turn, implies condition (15).

5   Dissimilarity generalized to non-identically routed schedules

We indicate here one way of generalizing the dissimilarity definition (3) to include the case when the schedule pair (σ_1, σ_2) is not identically routed.

First, note that the integration variable s in (3) can be normalized by the path length

$$L^{(\alpha)} = s^{(EXIT;\alpha)} - s^{(ENT;\alpha)};$$

i.e., introducing the new variable

$$\tilde{s} = \frac{s - s^{(ENT;\alpha)}}{L^{(\alpha)}},$$

one can rewrite the integral in (3) thus:

$$\int_0^1 \left[v^{(\sigma_1;\alpha)}(L^{(\alpha)}\tilde{s}) - v^{(\sigma_2;\alpha)}(L^{(\alpha)}\tilde{s}) \right]^2 d\tilde{s} \tag{19}$$

Suppose now that the two schedules assign to aircraft α two different paths, $\pi^{(\sigma_1;\alpha)}, \pi^{(\sigma_2;\alpha)}$, of respective lengths $L^{(\sigma_1;\alpha)}$ and $L^{(\sigma_2;\alpha)}$. The argument of each of $v^{(\sigma_1;\alpha)}, v^{(\sigma_2;\alpha)}$ in (19) can now be normalized by the corresponding path length, which gives the following generalization:

$$\int_0^1 \left[v^{(\sigma_1;\alpha)}(L^{(\sigma_1;\alpha)}\tilde{s}) - v^{(\sigma_2;\alpha)}(L^{(\sigma_2;\alpha)}\tilde{s}) \right]^2 d\tilde{s} \tag{20}$$

Finally, if there is a metric[5] ρ defined on the set of all paths in the given airspace, definition (3) can be generalized to include the case when the pair σ_1, σ_2 is not identically routed as follows:

$$d(\sigma_1, \sigma_2) =$$

$$\sqrt{\sum_\alpha \left\{ \rho(\pi^{(\sigma_1;\alpha)}, \pi^{(\sigma_2;\alpha)})^2 + \int_0^1 \left[v^{(\sigma_1;\alpha)}(L^{(\sigma_1;\alpha)}\tilde{s}) - v^{(\sigma_2;\alpha)}(L^{(\sigma_2;\alpha)}\tilde{s}) \right]^2 d\tilde{s} \right\}}$$

$$\tag{21}$$

Notice that if the schedule pair σ_1, σ_2 is identically routed, then

$$L^{(\sigma_1;\alpha)} = L^{(\sigma_2;\alpha)} \text{ and } \rho(\pi^{(\sigma_1;\alpha)}, \pi^{(\sigma_2;\alpha)}) = 0 \quad \text{ for all } \alpha,$$

and, consequently, (21) reduces to (3).

6 Conclusions

The above analysis helps make clear the distinction that, in the context of PATO, stability is a property of a parameter-dependent schedule, while robustness is a property of the execution of such a schedule. The intended use of a schedule dissimilarity metric, such as we have proposed above, is the evaluation of PATO robustness to operational uncertainties. The suitability of schedule dissimilarity (3) for modeling PATO system response to perturbations has not been established. Furthermore, models of perturbations to the parameters $p \in \mathcal{P}$ are necessary to

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[5]In the sense of [5, definition 12-5.2].

utilize dissimilarity in the operational context; the stochasticity of the input parameters may be difficult to test or false altogether. If schedule dissimilarity is suitable for PATO evaluation, and sufficient models of perturbations to the input parameters can be developed, a great deal of research remains before robust PATO can be realized. Considering the purposes for a quantifiable measure of schedule robustness stated in Section 3, either: an appropriate cost function for PATO schedule optimization must be developed, or the acceptable level of schedule sensitivity must be determined. Lastly, how such information might be employed operationally to increase PATO robustness must be investigated. While the amount of work remaining to realize the primary objective of this thread of research (robust PATO) is significant, this paper provides an important starting point for rigorous assessment of PATO scheduling methods in the presence of operational uncertainties.

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