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Separation-Compliant Time Advance in Terminal Area Arrivals: Tradeoff between Makespan and Fuel Burn

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The current operational practice in scheduling air traffic arriving at an airport is to adjust flight schedules by *delay*, i.e. a postponement of an aircraft's arrival at a scheduled location, to manage safely the FAA-mandated separation constraints between aircraft. To meet the observed and forecast growth in traffic demand, however, the practice of *time advance* (speeding up an aircraft toward a scheduled location) is envisioned for future operations as a practice additional to delay. Time advance has two potential advantageous capabilities: to increase the throughput of the arriving traffic, and to reduce the total traffic delay when the traffic sample is below saturation density. A cost associated with time advance is the fuel expenditure required by an aircraft to speed up. The contribution of this paper is an optimal control model of air traffic arriving in a terminal area that gives qualitative insight into the interdependence between two competing objectives: saving fuel (total, for the entire air traffic operation) and reducing the duration (makespan) of the operation. The numerical solution provided here was obtained using, for convenience, a software packaged based on the Pseudospectral Method with Legendre-Gauss-Radau collocation.

Nomenclature

$\dot{q}$	the time derivative of quantity q
A	the number of aircraft in a given traffic sample
$\mathcal{A}$	a finite set of A aircraft: $\mathcal{A} = \{1, 2, \ldots, A\}$
α	the index of an aircraft in the given sample: $\alpha \in \mathcal{A}$
$G = (V, E)$	a directed graph with vertex set V and edge set E
x^α	an arc length parameterization of the path of flight α in G
s^α	the instantaneous speed, dx^α/dt, of flight α along its path
w^α	the weight of the aircraft on flight α
a^α	the instantaneous acceleration, ds^α/dt, of flight α along its path
f^α or $f^\alpha(x^\alpha, s^\alpha, w^\alpha, a^\alpha)$	the negative of the instantaneous rate of fuel consumption for flight α (see (11))

I. Introduction

The current operational practice in scheduling air traffic arriving at an airport is to adjust flight schedules by *delay*, i.e. by a postponement of an aircraft's arrival at a scheduled location. This practice is aimed at managing safely the separation between pairs of aircraft when the system is near saturation. An alternative to delaying aircraft is speeding them up, a concept known as *time advance*. Time advance, although not in operational use today, is envisioned for future operations, in light of the observed and forecast growth in the intensity of Air Traffic Operations (ATO). The ability to practice time advance can reduce overall system delay in two types of situations: *i)* in the face of impending saturation, and *ii)* when the state of traffic alternates between saturation and under-saturation. A cost incurred by an aircraft exercising time

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advance is increased fuel expenditure. This calls for a model of the tradeoff between time advance and fuel consumption.

The contribution of this paper is such a model, constructed (section I.B) using optimal control theory,[1] and leading to a computational procedure for producing speed advisories. The model is similar in[2] but, by contrast, includes the inertia of the aircraft by treating the accelerations as the only control variables. The aforementioned tradeoff is analyzed numerically for one example (section III).

I.A. Background

A number of past research efforts have focused on the tradeoff between delay and fuel consumption.[2–4] An approach widely used in general Air Traffic Management (ATM) research is to cast the problem as a mixed-integer (non)linear program.[5–8] Since the sets of flights, waypoints, meter fixes, and route segments are finite, it is natural to think of the operational problems intuitively as discrete. The mixed-integer programming framework corresponds directly to this intuition, hence is a convenient model for capturing the problem realistically. In the context of modeling with a view toward future operational use, however, this framework faces the challenges of *i)* a lack of qualitative insight into the behavior of optimal solutions, and *ii)* absence of proofs of convergence or of bounds on the computational cost, or of both.

One advantage of optimal control over other areas of optimization is the availability of a general theory, notably the *Bellman Optimality Principle* and the *Pontryagin Maximum Principle.*[1] This theory offers insight into the qualitative behavior of optimal solutions, often before a solution is computed, and can yield computational solution procedures that allow for an analysis of *correctness* (i.e., the property that a computed result is in fact a solution to the problem) and of *low computational complexity bounds* (i.e., that the computation will complete in a low-degree polynomial time).[9,10] Mathematically provable presence of such properties is a highly desired feature for an algorithm that drives an automated tool for supporting safety-critical real-time ATM operations.

I.B. Model formulation

The model will rely on the following assumption.

Assumption I.1 The aircraft in question are flying in zero-wind conditions.

One may question whether the mathematical model developed here with the use of the simplifying Assumption I.1 is readily generalized to include reasonable uncertainties, such as wind. The role of Assumption I.1 in this study is as follows. Since an aircraft's navigation system will generally keep the aircraft on the prescribed path, the main effect of a wind field is from its component tangential to the path. The effect of uncertainties, such as wind or error in control execution, on the model is an introduction of error terms into the endpoints of the aircraft's speed range:

$$\begin{aligned} & \text{(min. speed)} + \text{(error due to uncertainty)} \\ \leq\ & \text{(the aircraft's ground speed)} \\ \leq\ & \text{(max. speed)} + \text{(error due to uncertainty)} \end{aligned}$$

The mathematical setting of this paper, aimed at modeling a set of aircraft arriving in a terminal airspace, is as follows (see Refs.[2,11,12] for a more detailed exposition). The airspace graph is modeled as a *directed graph*, or *digraph* [13, section A.2], $G = (V, E)$, where V is the vertex set, and E the edge set. Each vertex $v \in V$ is a point in a Euclidean space $\mathbf{E}$ of dimension 2 or 3. If $e \in E$ is an edge from vertex v_1 to vertex v_2, then the nominal route segment from waypoint v_1 to waypoint v_2 is a curve in $\mathbf{E}$, connecting v_1 to v_2. All such curves will henceforth be assumed *rectifiable*[14] and thus capable of an arc length parameterization. A cusp in the curve can be approximately smoothed if inertia is to be (as it is in this paper) taken into account. A graph-theoretic *path*[13] in G is, therefore, associated (and, henceforth, identified) with a spatial path that can be traversed by a aircraft. A vertex of G of *indegree*[13] ≥ 2 (resp., outdegree ≥ 2) corresponds to two or more route segments merging (resp., diverging). The modeling framework below imposes no restrictions on the outdegree or indegree of a vertex. A graph $G = (V, E)$ with this additional geometric setting (known in computational geometry as a *graph embedding*) will be called *an airspace graph.* This model of a terminal airspace is not part of today's Air Traffic Operations, but rather is envisioned in NextGen, as the following quote from Ref.[15] explains:

> ...while standard published routes have been specified in the en-route airspace, air traffic procedures used in terminal airspace seldom specify a continuous route of flight to the aircraft destination or to the exit from terminal airspace. From Top of Descent (i.e., the completion of the cruise stage of flight) to the start of the final approach to landing, arriving flights today navigate by procedures rather than by a fixed set of routes. Arrival and departure procedures may include constraints on speed and altitude, but usually do not specify a continuous route from Top of Descent to the destination airport (or from the origin airport to cruise altitude for departing aircraft).

The arriving flights are indexed by the finite set of the first A positive integers:

$$\mathcal{A} = \{1, 2, \ldots, A\},$$

where A is the number of flights.

The above construct of airspace graph allows to model the route of a flight $\alpha \in \mathcal{A}$ as a *walk*[13] in G (i.e., a finite sequence of vertices, not necessarily distinct, with each consecutive pair an edge in G) and to furnish this route–which is, by the above, a rectifiable curve in a 2- or 3-dimensional Euclidean space $\mathbf{E}$, which models the physical airspace–with an arc length coordinate x^α. Thus, a motion of flight α along its path is completely specified by a parametrization $x^\alpha(t)$ of the arc length coordinate by time, t, and at each time instant t the arc length coordinate value $x^\alpha(t)$ completely specifies the physical location (denoted $x^\alpha_{\mathbf{E}}(t)$) of aircraft α in $\mathbf{E}$.

If $||\cdot||$ is the Euclidean norm on $\mathbf{E}$, then the minimal separation requirement for aircraft α_1 and α_2 acquires the form

$$||x^{\alpha_1}_{\mathbf{E}}(t) - x^{\alpha_2}_{\mathbf{E}}(t)|| \geq (\text{a minimal separation distance}), \quad \text{for all valid } t. \tag{1}$$

In general, the right-hand side of the inequality in (1) is a function of $x^{\alpha_1}_{\mathbf{E}}(t), x^{\alpha_2}_{\mathbf{E}}(t)$, since the required minimal distance can depend on the relative positions of the two aircraft and on the specific portion of the airspace occupied by them. Conditions (1) enforce pairwise aircraft separation *continuously in time*, as is required operationally by the FAA.[16]

All aircraft are assumed here to enter the model simultaneously, at time $t = t_0$, at the prescribed initial locations and speeds, and with the prescribed initial weights,

$$x^\alpha(t_0) = x^{INIT;\alpha}, \quad s^\alpha(t_0) = s^{INIT;\alpha}, \quad w^\alpha(t_0) = w^{INIT;\alpha}, \quad \alpha \in \mathcal{A}, \tag{2}$$

and that aircraft α exits the model by reaching the prescribed destination, $x^{DEST;\alpha}$, at a time $t^{ARR;\alpha}$ of arrival at destination. The arrival times, unknown at the outset, must be found as part of solving the optimal control problem formulated below (problem I.1), but the order in which the aircraft arrive, each at its destination, is prescribed[a]. Thus, the indices α can be arranged in a sequence $\alpha_q,\ q = 1, \ldots, A$, such that

$$t^{ARR;\alpha_q} \leq t^{ARR;\alpha_{q+1}} \quad \text{for } q = 1, \ldots, A-1, \tag{3}$$

and the arrival requirement can be written as the system

$$x^\alpha\left(t^{ARR;\alpha_q}\right) = x^{DEST;\alpha_q}, \quad s^\alpha\left(t^{ARR;\alpha_q}\right) = s^{DEST;\alpha_q}, \quad q = 1, \ldots, A, \tag{4}$$

where the equations specifying the arriving speeds are optional[b] and can be either omitted or recast as inequalities that constrain the arrival speed to lie in a given interval.

Remark I.1 *The difference*

$$t^{ARR;\alpha_A} - t_0 \tag{5}$$

is the duration of time that elapses from the start of the air traffic operation to the latest arrival. This duration is known in ATM as the makespan.[17]

[a]This prescription substantially restricts the space of candidate solutions. Even without this restriction, however, there exist instances of the problem that have no feasible solution.

[b]One situation where the exit constraints on the speeds are needed is when the aircraft are landing: they must land approximately at landing speed.

Another requirement that will be imposed on the air traffic operation modeled here is the *makespan requirement*,

$$t^{ARR;\alpha_A} - t_0 \leq M, \tag{6}$$

that the makespan not exceed a pre-assigned value M (which specifies the amount of time allocated for the operation).

In what follows, let $s^\alpha(t)$ denote the instantaneous airspeed of flight α along its path, and $a^\alpha(t)$ the acceleration, i.e.

$$\left.\begin{array}{rcl} \dot{x}^\alpha & = & s^\alpha \\ \dot{s}^\alpha & = & a^\alpha \end{array}\right\} \quad \text{for all flights } \alpha \text{ that have not yet reached destination.} \tag{7}$$

The weight w^α of aircraft α will be modeled–like the x^α's, s^α's, and a^α's–as a time-dependent variable that evolves according to equation (11), derived in section II, below.

Operationally, s^α is restricted to a feasible range,[18]

$$s^{MIN;\alpha} \leq s^\alpha \leq s^{MAX;\alpha}, \quad \alpha \in \mathcal{A}. \tag{8}$$

In the control problem formulated below (problem I.1), the quantities x^α, s^α, and w^α will play the role of the state variables; the variables a^α, the role of the control variables.

As the aircraft execute a given control strategy

$$\mathbf{a}(t) = \left(a^1(t), \ldots, a^A(t)\right),$$

the corresponding state trajectory

$$\left(\mathbf{x}(t), \mathbf{s}(t), \mathbf{w}(t)\right),$$

where

$$\mathbf{x}(t) = \left(x^1(t), \ldots, x^A(t)\right), \quad \mathbf{s}(t) = \left(s^1(t), \ldots, s^A(t)\right), \quad \mathbf{w}(t) = \left(w^1(t), \ldots, w^A(t)\right),$$

is a solution of the initial value problem (2), (7), (11). In this execution, aircraft α will be assumed to incur fuel consumption at the instantaneous rate $-\dot{w}^\alpha$, defined by equation (11), below, and measured in *Newtons per second*, so that the total weight (in Newtons) of the fuel consumed by flight α is

$$w^\alpha(t_0) - w^\alpha\left(t^{ARR;\alpha}\right)^{\text{c}}. \tag{9}$$

Treating the required arrival times $t^{ARR;\alpha}$ as optimization parameters, one can explore the tradeoff between throughput and fuel consumption by studying the following problem in optimal control:

Problem I.1 *Find a control strategy* $\dashv(t)$ *and the corresponding state trajectory* $(\mathbf{x}(t), \mathbf{s}(t), \mathbf{w}(t))$ *that maximize the sum of the aircraft weights upon arrival,*

$$\max \rightarrow \sum_{\alpha \in \mathcal{A}} w^\alpha\left(t^{ARR;\alpha}\right), \tag{10}$$

and satisfy the following constraints:

- *the initial condition (2),*
- *the system (7), (11) of* state equations,*[14]*
- *the arrival condition (4),*
- *the makespan requirement (6),*
- *the state constraints (1) (the minimal separation requirement which, for the purposes of solving this problem will be rewritten in terms of the arc length coordinates* $x^{\alpha_1}, x^{\alpha_2}$*–see Refs.[11, 12] for detailed calculations),*
- *the state constraints (8) (feasible range of airspeed), and*
- *the parameter constraints (3) (order of arrivals).*

[c] The total fuel consumption expressed here by formula (9) was used in Ref.,[2] as an integral of $\dot{w}^\alpha$ with respect to time, incorrectly. The error–pointed out by M. G. Wu–was the omission of the last occurrence of $s^\alpha(t)$ in the integrand.

II. A differential equation describing the instantaneous rate of fuel consumption

The differential equation mentioned in the title of this section will be derived for a flight α which has not yet reached destination. This derivation is based on a number of simplifying assumptions and may not carry the fidelity of the other, considerably more complicated, models, such as that in Ref.[19] The parameters and notation used in what follows are listed in Table 1.

symbol	**meaning**
g	acceleration due to gravity
k	the constant of proportionality in the relation between the coefficients of lift and drag [20, equation (11-5)]
θ_{attack}	the angle of attack
θ_{path}	the flight path angle (the angle between the direction of descent and the horizontal)
$\mathcal{S}$	the reference area for the aircraft's angle of attack
m^α or $m^\alpha(t)$	the mass of aircraft α at time t
w^α or $w^\alpha(t)$	$g\, m^\alpha(t)$
TSFC	thrust-specific fuel flow (mass per hour per unit force)
d_{air}	air density
T	thrust

Table 1: Model parameters defined.

Quantity d_{air} is a linear function of the altitude, hence of x^α.

The derivation will be based on the following assumption.

Assumption II.1 *(a) The surface of the Earth underlying the descent is flat, "horizontal."*

(b) The flight path angle, θ_{path}, is constant[d]*. (Consequently, the path is rectilinear, making an angle of θ_{path} with the horizontal plane.)*

(c) The angle of attack, θ_{attack}, is constant. (Consequently, so is $\mathcal{S}$.)

The rate of fuel consumption will be expressed in terms of the acceleration a^α and of a number of other quantities, using two equations. The first is the bottom equation on page 100 of Ref.,[20] which we reproduce here, but in our notation and without the assumption that $\cos(\theta_{attack})$ is approximately equal to 1:

$$\dot{s}^\alpha = g\left(\frac{T\cos(\theta_{attack}) - D}{w^\alpha} - \sin(\theta_{path})\right).$$

The second is the formula (see [20, page 151])

$$T = \frac{\dot{m}}{\text{TSFC}} = -\frac{\dot{w}^\alpha}{g\ \text{TSFC}}$$

that connects thrust to fuel flow (both positive quantities). Ultimately, one obtains

$$\dot{w}^\alpha = f^\alpha\left(s^\alpha, w^\alpha, a^\alpha\right) = \frac{TSFC\ w^\alpha}{\cos(\theta_{attack})}\left(-g\ \sin(\theta_{path}) - \frac{2g\ k\ w^\alpha}{d_{air}\ (s^\alpha)^2\ \mathcal{S}} - a^\alpha\right). \tag{11}$$

With the parameter values specified in the top seven rows of Table 2 and with the values (in MKS)

$$s^\alpha = 82.0, \quad w^\alpha = 4.9 \times 10^5, a^\alpha = -0.1$$

the right-hand side of (11) takes the value that corresponds to the fuel consumption rate of 0.4 kg/s, which is consistent in the order of magnitude with the values shown in [19, Figure 12].

[d] This assumption implies that the resultant force acting on the aircraft is directed along the flight path and has zero component transverse to the path.

III. A sample numerical solution

The following instance of optimal control problem I.1 was solved with M varying from 571.82 to 569.42 in steps of 0.1. The solutions were obtained using the GPOPS[21] (version II) numerical software package, which uses the Pseudospectral Method with Legendre-Gauss-Radau collocation.

III.A. Assumptions and the numerical values of the parameters used

The transportation network appearing in the numerical examples of this section is a graph; namely, no two vertices are connected by more than one edge. This allows the specification of each path as a sequence of vertices, rather than of edges. To simplify the computations, the separation requirements for each pair of aircraft are assumed symmetric. The numerical code admits a straightforward, albeit somewhat cumbersome algebraically, generalization that will dispense with this assumption. All plots were generated using the Matlab software.[22] The units used in the computations were MKS (SI units). The numerical values used for the parameters (including those parameters defined in Table 1) are listed in Table 2. They were chosen to represent approximately the descent functionality of the later models of the Boeing 737 series.

parameter	**value**	**units**
g	9.8	m/(s^2)
k	0.1	(dimensionless)
θ_{attack}	0.1	rad
θ_{path}	-0.05	rad
$\mathcal{S}$	120.0	m^2
TSFC	2.0×10^{-5}	kg /(hr N)
d_{air}	1.225	kg / (m^3)
minimal separation	3.0	NMI
initial mass of each aircraft (MKS)	5.0×10^4	kg
initial weight of each aircraft (MKS)	4.9×10^5	N
initial speed of each aircraft	160.0	kts
initial speed of each aircraft (MKS)	82.311	m/s
minimal allowed speed	120	kts
minimal allowed speed (MKS)	61.7333	m/s
maximal allowed speed	200.0	kts
maximal allowed speed (MKS)	102.8888	m/s
minimal allowed acceleration (MKS)	-0.5	m/(s^2)
maximal allowed acceleration (MKS)	0.5	m/(s^2)

Table 2: Model parameter values specified.

The geometry of the terminal airspace (shown in Figure 1) resembles a portion of the Dallas-Fort Worth Airport (DFW).

The routing and sequence of arrival at destination (each aircraft's destination is the last vertex in its route) are as follows:

```
aircraft_to_path_assignment{1}.path = [9 11];
aircraft_to_path_assignment{2}.path = [4 9 11];
aircraft_to_path_assignment{3}.path = [8 10 12];
aircraft_to_path_assignment{4}.path = [7 8 10 12];
aircraft_to_path_assignment{5}.path = [3 4 9 11];
```

The numerical solutions are plotted in Figures 2 and 3.

Figure 1: The instance considered in section ??: the initial positions of the aircraft.

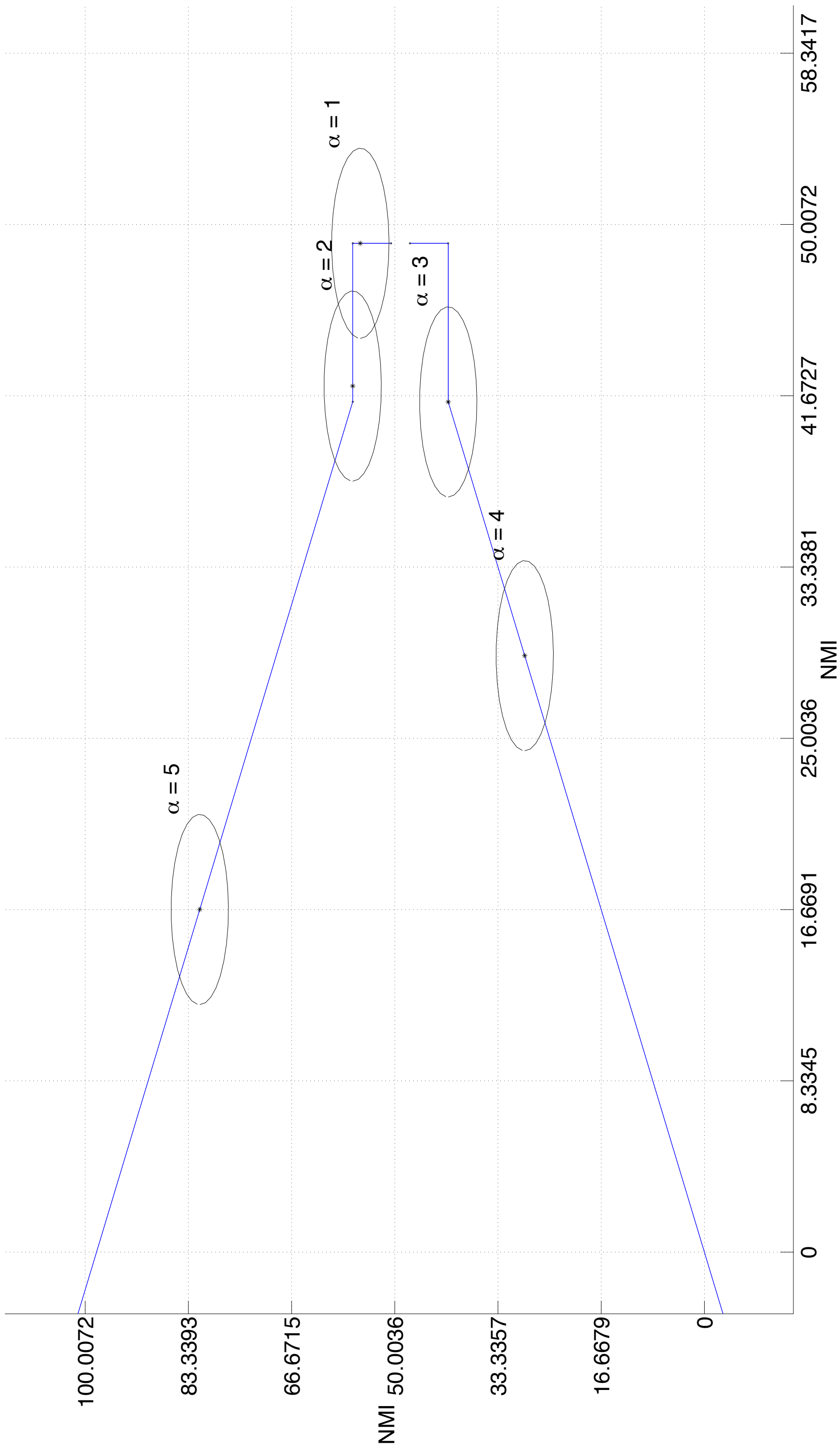
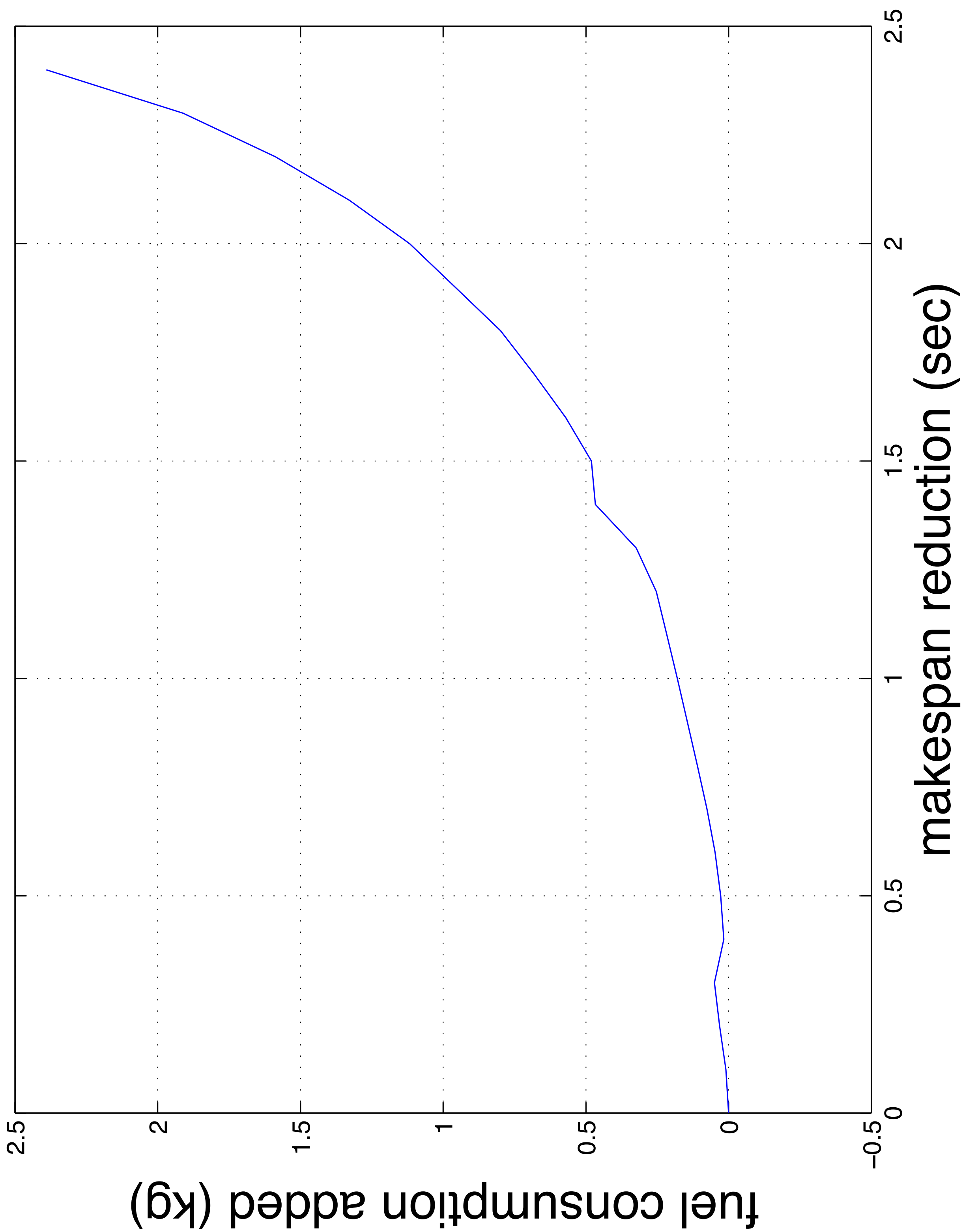
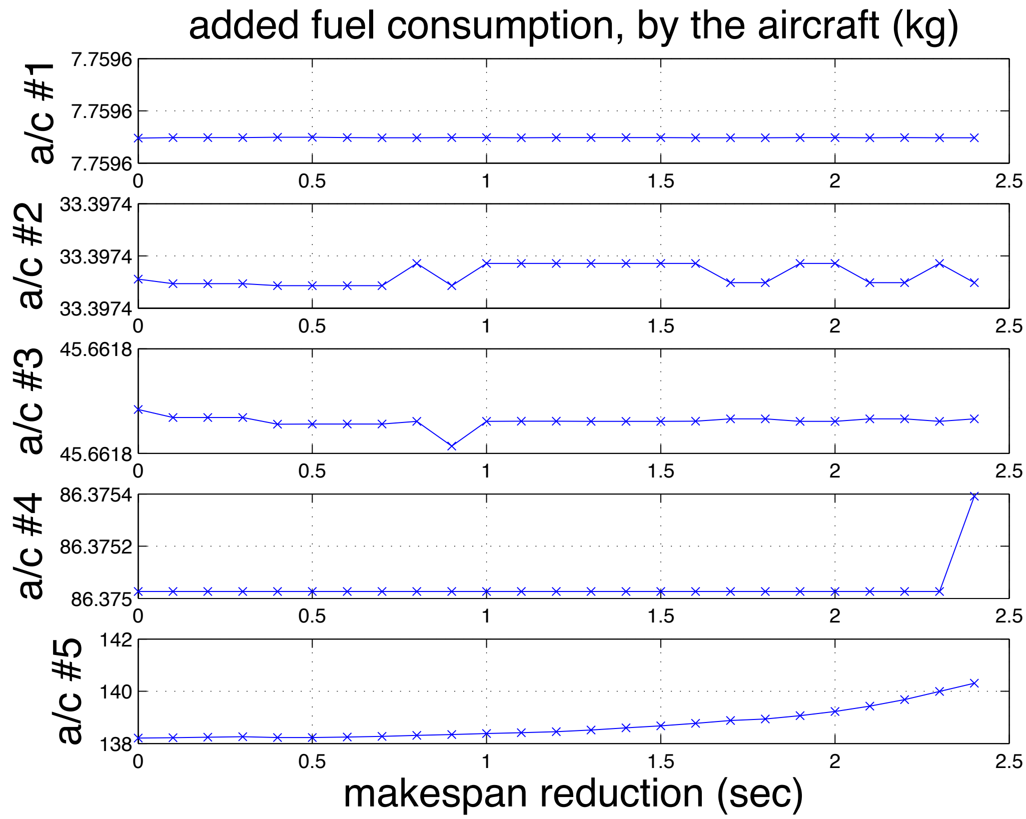
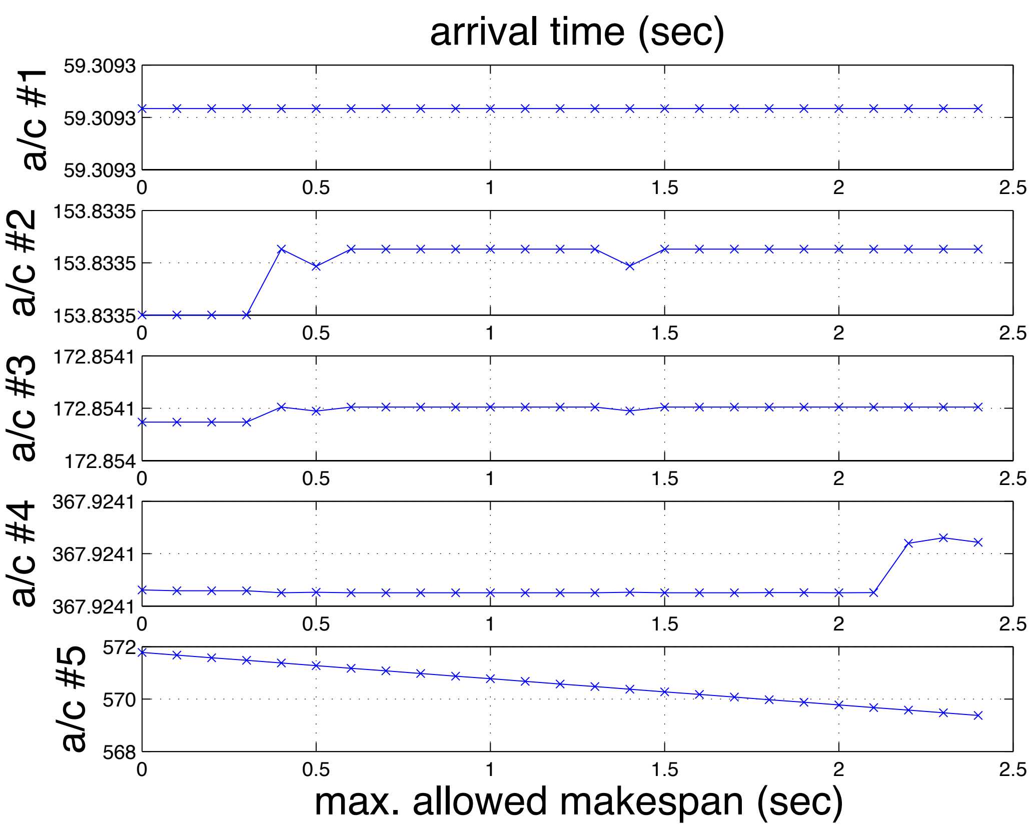


Figure 2: The increase in fuel consumption (by aircraft) incurred by constraining makespan vs. makespan reduction.





(A)



(B)

Figure 3: (A) The increase in individual (by aircraft) fuel consumption incurred by constraining makespan vs. makespan reduction. (B) The individual arrival times of the aircraft vs. makespan reduction.

IV. Discussion

The numerical results above support the following intuition:

- an optimal control (acceleration) strategy is the same for all those values of M which make constraint (6) inactive, and
- once M decreases beyond a certain threshold value (which depends on the specific airspace, on the initial state aircraft traffic, and on the other constraints), total optimal (here, minimal) fuel consumption increases as M decreases.

The results suggest, further, that in the optimization problem formulated as above, a decrease in M affects mainly those aircraft that have no others behind. Introducing an additional cost term for passenger comfort (e.g., in the form $\sum_\alpha \int_{t=0}^{t^{ARR;\alpha}} a^\alpha(t)^2\, dt$), however, may distribute the fuel expenses among the aircraft more evenly.

The plot in Figure 1 shows a central tradeoff of time advance: fuel consumption added vs. makespan time saved.

Numerical analysis of a wider variety in airspace geometry and in aircraft initial positions is a direction of further research. Other directions for further research include adding constraints that increase the fidelity of the model; e.g., constraining the landing speed to an appropriate and narrow range, and adding a running cost term that reflects the loss of passenger comfort at large acceleration.

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