

An Approach for Balancing Delay and Fuel Burn in Separation Assurance Automation

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A conflict resolution algorithm is introduced that enables a user to specify the degree to which fuel economy is prioritized relative to airborne delay, analogous to the “cost index” setting in flight management computers. Fast-time simulations of current-day traffic levels in two regional airspaces under nominal weather conditions are simulated to evaluate the benefit of modifying a conflict resolution algorithm to select resolution maneuvers based on minimum cost. The study employs the use of a parameter to represent the relative importance of fuel burn price to delay price. Additionally, the price of fuel burn and delay relative to one another is varied from the nominal in order to illustrate the differences in the algorithmic behavior should the delay or fuel price increase. Results show the lowest total operational cost occurs when the relative importance of delay to fuel burn price is equally weighted. Overall, minimizing fuel burn when selecting resolutions based on cost results in a lower operational cost than minimizing delay. The sensitivity of total operational cost to resolution types is presented. Implications of these findings for advanced separation assurance concepts are discussed.

I. Introduction

AIR traffic demand is projected to increase significantly in the upcoming years.¹ The human workload associated with conflict detection and resolution is expected to limit this increase and thereby limit the economic growth that aviation facilitates. Automated separation assurance systems are proposed as a way to safely and efficiently separate aircraft in highly dense traffic conditions up to two to three times current levels. Numerous algorithms have been proposed to provide separation assurance in the future air traffic system.^{2,3} Maintaining safe separation is the first-order objective of all such algorithms; however, the second-order objectives can vary and are the focus of much research in the field of air traffic management. With any automated resolution tool, the resolution selected is based on some criterion function. The majority of the proposed algorithms optimize the selection of conflict resolution maneuvers to minimize airborne delay in order to mitigate the effect on schedule. However, the true cost of operations is more complex with considerations beyond delay. With the tremendous rise in fuel price over the past few years, examination of the implications of fuel price has increased in relevancy. A additional objective, then, is to optimize based on fuel burn. Prior research, as in Ref. 4, showed that the minimum-delay solution was rarely the same as the minimum-fuel-burn solution.

In Ref. 4, the system performance of a conflict resolution algorithm that selected resolutions based on minimum-delay was compared to the system performance of the same algorithm when selecting resolutions based on minimum-fuel-burn. The most effective maneuver when minimizing for fuel burn was a speed reduction maneuver, which employs a temporary speed reduction to resolve the predicted conflict. However, speed reductions were selected less frequently than other, less fuel-efficient maneuvers. Additionally, when utilized, speed reduction maneuvers significantly increased the cumulative delay. When selecting resolutions based on minimum-fuel-burn, a 40% reduction in fuel burn was realized as compared to the the conventional minimum-delay approach. However, the delay incurred with those more fuel-efficient resolution maneuvers was nearly twice that observed with the minimum-delay approach. The stark contrast suggests there could be value to an approach that considers the cost of both delay and fuel burn.

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The desire to balance the costs of delay and fuel burn is evident in today's Flight Management Systems (FMS). Most airlines use a ratio of the two costs to determine the economy speed for a given flight on a given day. This ratio is called the Cost Index, and it determines the "economy" speed profile for a flight by minimizing the total cost of operation. The Cost Index is the ratio of the time-related operating costs of the aircraft vs. the cost of fuel. This process can be applied when determining how best to resolve a conflict. Where previous studies have explored resolution selection based on minimizing either delay or fuel burn, the algorithm in this study was modified to minimize cost given a parameterized expression of their relative importance. This study examines the system performance of a conflict resolution algorithm capable of selecting maneuvers based on minimum cost in realistic traffic scenarios.

The paper is organized as follows. Section II presents the cost function that considers the price of fuel and the "price" of airborne flight delay. The conflict resolution algorithm into which this cost function was embedded is described in Section III. Section IV sets forth the experiment design, and Section V presents the results. A summary of the conclusions and discussion of future work conclude the paper.

II. Balancing Delay and Fuel Burn

This study evaluates the performance of modifying the resolution selection method to evaluate fuel and delay together. An approach to accomplish this is to design a cost function that normalizes delay and fuel burn. Since delay and fuel burn produce a cost in a true sense of the term, the monetary amount of resolutions is modeled within our system and used as the optimization criteria in our resolution selection scheme. Furthermore, a mechanism to vary the weight of delay and fuel burn in a cost function creates a separation assurance feature similar to the Cost Index of a FMS. This section describes an approach for quantifying the cost of a resolution as a function of delay and fuel burn, and selecting the least-cost resolution for a given conflict.

Often, the cost of delaying a flight differs from flight to flight. This cost can be most accurately estimated by the airlines; unfortunately these models are not regularly available. In Ref. 5, the cost of different types of delay (airborne, ground, etc.) were estimated for an array of aircraft sizes. The average airborne delay price of \$20.00 per minute, for passenger aircraft of 100 seats or more, was used as the nominal delay price for this study. Since most conflict resolutions produce delays of less than a minute, delay price translated to \$0.33 per second. Present-day fuel price of approximately \$0.43 per pound was used for resolution selection. The structure of the employed conflict resolution algorithm allows for the tabulation of both delay and fuel burn for each resolution considered. Using these metrics, a cost function descriptive of the relationship between the price of fuel and the amount of fuel and the price of delay and the amount of delay was derived. Equation (1) describes the operational cost:

$$C_O = (FB \times P_{FB}) + (D \times P_D) \quad (1)$$

where FB is the fuel burn in pounds, P_{FB} is the fuel price in dollars per pound, D is the delay in seconds and P_D is the price of delay. For the purpose of this study a resolution cost function, C_R , was developed. The relative importance of delay to fuel burn within C_R is represented by the inclusion of a user specified weight parameter: alpha. In this scenario, the user represents the Federal Aviation Administration (FAA). The availability of the alpha parameter could allow the FAA to balance system wide preferences for delay and fuel burn thus allowing national optimization of the air traffic control system. For example, the user could shift alpha to favor fuel burn savings when flights are not constrained by time (i.e. a flight is early and would otherwise be delayed because of traffic flow management or the gate is not ready) and shift to delay for aircraft that need to be scheduled more closely. The range of alpha is shown in Eq.(2):

$$0 \leq \alpha \leq 1 \quad (2)$$

Using alpha to represent the weight of a given parameter to another, the resolution cost [Eq.(1)] can be expressed as Eq.(3):

$$C_R = [\alpha(D \times P_D) + ((1 - \alpha) \times (FB \times P_{FB}))] \quad (3)$$

where $\alpha=0$ represents minimum fuel burn optimization and $\alpha=1$ is minimum delay optimization. The calculation of fuel burn and delay is discussed in Section IV.E.

For this study, resolution cost, C_R , is used as the criteria in which the optimal resolution is selected to resolve a conflict. The results of this process is controlled by the user-specified alpha value based on the importance of fuel burn and delay. Since resolution cost is a theoretical term, in later sections, operational cost, C_O , is analyzed to represent the actual price to the airspace users.

III. Implementation

A. Advanced Airspace Concept Autoresolver

The Advanced Airspace Concept Autoresolver (AAC Autoresolver) is a strategic conflict resolution algorithm designed to deconflict aircraft that are predicted to lose separation more than two minutes in the future. The Autoresolver resolves aircraft conflict pairs ordered by predicted time to first loss of separation. For each conflict in the conflict list, the Autoresolver follows an iterative approach for resolution. These trajectories take into account characteristics such as aircraft type, speed and airspace boundaries. The Autoresolver calculates future trajectories composed of waypoints, speeds and altitudes which may possibly resolve the conflict.

Figure 1 shows the types of trajectory changes attempted by the Autoresolver grouped in terms of horizontal, vertical, or speed maneuvers. The dashed lines in Figure 1 indicate the suggested trajectory changes to avoid the predicted conflict. This trajectory change is then sent to a trajectory engine that computes a corresponding trial resolution trajectory.

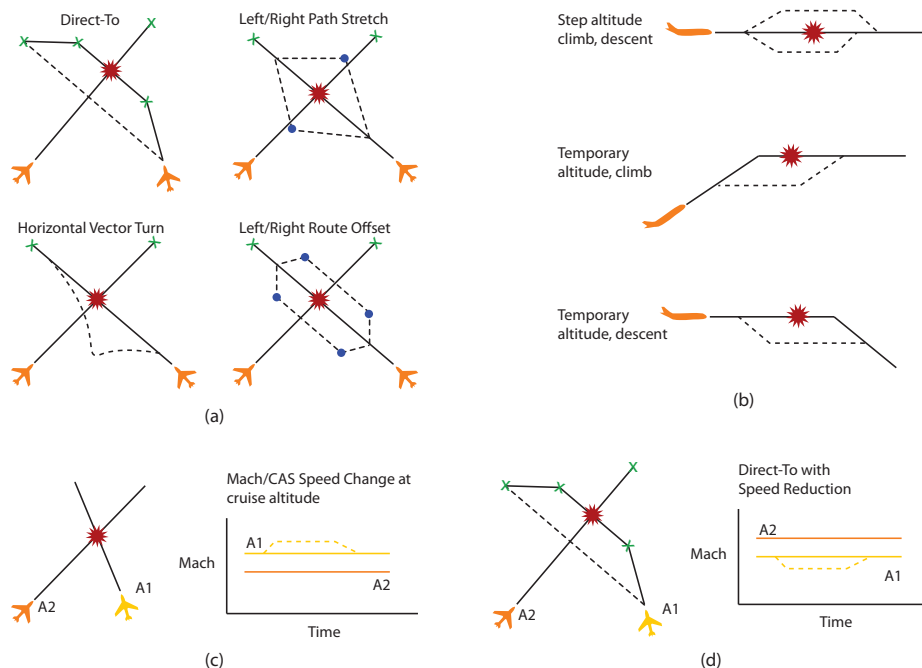


Figure 1: Resolution trajectories of type horizontal (a), vertical (b) and speed (c & d)

A resolution trajectory is considered viable, successful (and stored), if it resolves the primary conflict, and is free of predicted losses of separation with all aircraft for a specified period of time. If the trial resolution is not conflict free, the Autoresolver computes a new trial resolution and checks if it is successful. For each resolution type this iteration is continued until a successful resolution is found or all possibilities of that type have been tried. For each successful resolution, both the associated delay and the fuel burn are calculated.

The Autoresolver will generate up to 18 successful resolutions per aircraft in conflict for a total of up to 36 between the two aircraft. In this study, the algorithm selected a resolution from among the set of successful resolutions by calculating the cost per resolution and selecting the resolution with the lowest cost. The selected resolution was then implemented via fast-time, closed-loop experiment as discussed in the following sections. Using the equation formulated in the previous section, the result of the AAC computations is a list

of resolutions and their associated costs. Further discussion regarding the design of the algorithm and the types of resolutions that are generated is presented in Refs. 6, 7.

IV. Experiment Design

This section describes the fast-time simulation environment, test parameters, and the metrics used in the study.

A. Simulation Environment

The Airspace Concept Evaluation System (ACES) is a fast-time, agent-based simulation of the National Airspace System (NAS) that uses four-degree-of-freedom (4 D.O.F) equations of motion based on the Base of Aircraft Data (BADA) to generate aircraft trajectories.⁸ ACES was developed specifically to provide a general purpose environment for evaluating future air traffic management and control concepts, including automated resolution algorithms. Each flight's trajectory is simulated from the departure fix associated with its original airport and ends at the arrival fix associated with its destination airport. By using aircraft-type-specific performance data together with guidance and navigation models, the ACES trajectory engine can generate representative trajectories for many aircraft. For the purposes of this study, the aircraft trajectories were entirely deterministic with no trajectory uncertainty. Aircraft conflicts were predicted with perfect accuracy, and resolution trajectories were guaranteed to be followed precisely by the simulated aircraft. In addition to deterministic aircraft trajectories, certain simplifications were made in the modeling and execution of the experiment: negotiation of resolution trajectories between aircraft operators and/or the air navigation service provider were not modeled, and neither data link transmission delays nor pilot-action delays were modeled. Once a resolution trajectory was selected by the automation it was executed immediately and precisely.

B. Airspace and Traffic

In this study, the Autoresolver resolved roughly 1,885 conflicts in two Air Route Traffic Control Centers (ARTCCs): Indianapolis (ZID) and Chicago (ZAU). The ARTCCs selected contain primarily cruising traffic which is of interest as this study focused on the resolution of en route conflicts. Flight operations over a 6-hour period were simulated based on Aircraft Situation Display to Industry (ASDI) data recorded March 8, 2007 which represented a "low weather," high volume day in the NAS. The data set included 23,000 flights of varying types, their associated routes, and their departure times. The Rapid Update Cycle wind data was used to model winds in the selected ARTCCs. Figure 2 shows a subset of the ARTCCs in the central region of the United States with ZID and ZAU shaded.

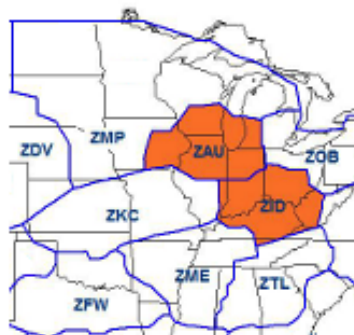


Figure 2: The ARTCCs studied in this experiment.

C. Test Matrix

Table 1 shows the test matrix used in this study to investigate the benefits of selecting conflict resolution maneuvers based on minimum cost. The matrix includes two independent variables: alpha and price index.

Nine test points were chosen for alpha evenly distributed between 0 and 1 at 1/8 increments. Three test points were chosen for price index: Nominal, Double the fuel price, and Double the cost of delay. “Nominal” represents a fuel price and delay costs at current-day values. Double the Fuel Price and Double the Delay Price describe test points for which P_{FB} or P_D is doubled, respectively.

Table 1. Experiment Factors and Levels.

Experiment Factors	Levels
Optimization	Resolution Cost
α	0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.875, 1
Price Index	Nominal, Double the Fuel Price, Double the Delay Price

D. Dependent Variables

Three metrics were selected for comparison: the number of conflicts per flight hour, delay and fuel burn. A conflict is said to occur when two aircraft are predicted to come within 5 nautical miles horizontally and 1,000 feet vertically from each other some time in the future (i.e. 20 minutes). The flight hour metric is calculated by summing the total flying time within ZAU and ZID of every flight in the simulation. In the study, the number of conflicts per flight hour is used as a proxy for complexity. The delay metric is defined as the additional delay incurred per resolution, in seconds, as compared to the original (i.e., conflicted) trajectory. The fuel-burn metric is defined as the additional fuel burned per resolution, in pounds, as compared to the original trajectory. Fuel burn is modeled as a function of thrust, true airspeed, and altitude using BADA.

E. Delay and Fuel Price Parameters

The cost of airborne delay used in this study was approximated from the values for airborne delay costs presented in Table 4 of Ref. 5 for passenger aircraft greater than 100 persons. The operational data used to generate the cost figures in the referenced study were collected from European Airlines. A summary of the cost of delay to airlines during various trip segments is presented in Ref. 9. The fuel price used in this study was taken from the International Air Transport Association (IATA) Jet Fuel Price Monitor.¹⁰ This study used a price from August 2012 to represent the Nominal Cost Index. The delay and fuel prices for the different Price Index levels used in this study is presented in Table 2.

Table 2. Delay and fuel burn prices for the various Price Index levels.

Price Index	Delay Price (\$/seconds)	Fuel Price (\$/pound)
Nominal	0.33	0.43
Double the Fuel Price	0.33	0.86
Double the Delay Price	0.66	0.43

V. Results

This study evaluates the effects of a cost-based resolution selection criterion on system efficiency. Metrics for complexity and cost are examined to quantify the impact of modifying the AAC Autoresolver. The cost metrics and resolution-type cost results are presented parametrically in terms of the delay cost and fuel burn cost of selected resolutions. The cost associated with various resolution types is investigated, and their implications are discussed.

A. Complexity

In order to assess how the resolution selection criterion (alpha) affects the complexity of the conflict resolution problem generally, the number of conflicts per flight hour was examined. The number of conflicts per flight

hour provides insight into the algorithm's response to the inclusion of cost-based resolution selection. A significant increase in the number of conflicts per flight hour, as a result of including the minimum-cost resolution selection approach, might suggest an increase in problem complexity. Figure 3 shows that using cost-based resolution selection does not significantly increase the observed number of conflicts per flight hour. The small increase in the number of conflicts per flight hour observed between $\alpha=0$ and $\alpha=1$ may be a by-product of the resolution selection process. In *Bowe et al.* (Ref. 4) minimum-delay resolution selection was found to favor timesaving maneuvers such as route shortcuts. When selecting resolutions based on minimum fuel burn the algorithm displayed a preference for speed reduction maneuvers which were shown to increase the cumulative delay.

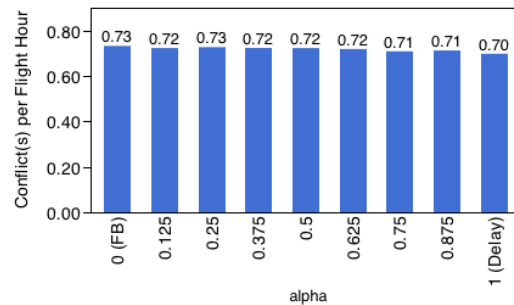


Figure 3: Conflicts per Flight Hour vs alpha.

B. Costs Associated with Balancing Delay and Fuel Burn

Most flight management systems in operation today have configurable cost index settings to select the most efficient speed profile according to the users' needs. This allows the user to weigh the importance of saving time or saving fuel per flight. For example, if a given flight is ahead of schedule, and connecting flights are in question, a user may change the cost index in the FMS to favor fuel savings. Likewise, if a flight is behind schedule the user could change the index to increase the importance of delay. Similar to this cost index paradigm for favoring delay or fuel burn in certain cases, the results of this experiment can be modeled in a way that allows one metric to be weighed more heavily than the other.

Figure 4(a) shows the cost as a function of alpha. As expected, when $\alpha=0$ the fuel burn cost is minimized, and when $\alpha=1$ delay cost is minimized. When $\alpha=1$ the contribution of the fuel cost to the operational cost equation is zero but each resolution still has an associated amount of fuel burn. Conversely, in the minimum fuel burn case, the total cost is dominated by the delay cost. The lowest total operational cost roughly occurs when $\alpha=0.5$. The figure reveals the minimum delay case to be the most expensive overall, as reflected by the Total Operational Cost curve, which is primarily dominated by the fuel cost. This result suggests that optimizing conflict resolution maneuvers for minimum delay may be the least cost effective approach.

Figure 4(b) shows the total operational cost as a function of alpha for the three price indices. The observed trends suggest that, when the cost of delay is doubled, the overall operational cost is higher than when the fuel price is doubled, with the exception of when alpha is between 0.875 and 1. As expected, the nominal price index produces the lowest total operating cost and the least dramatic fluctuation in cost over the range of alpha, until alpha is greater than 0.875.

Of interest in Figure 4(b) is the increase in total operational cost when the price of delay is doubled and $\alpha=0.5$. Further investigation revealed a large disparity in the total operational cost for en route and arrival conflicts for this simulation setting. An arrival conflict is defined when a maneuvered aircraft is predicted to conflict with another within 20 minutes of its arrival fix. All other instances of conflict are considered en route. Figure 5 shows the total operational cost when the price of delay is doubled for en route and arrival conflicts. The rise in operational cost is directly due to the spike in operational cost for arrival conflicts when $\alpha=0.5$. Results show that when delay and fuel costs were evenly balanced in the Double the Delay Price simulation runs, a large amount of speed reductions were selected as the optimal cost-based resolution, thus creating arrival sequencing congestion where additional resolutions were required to separate the flow. More analyses are needed to evaluate the impact of optimizing cost when resolving arrival conflicts.

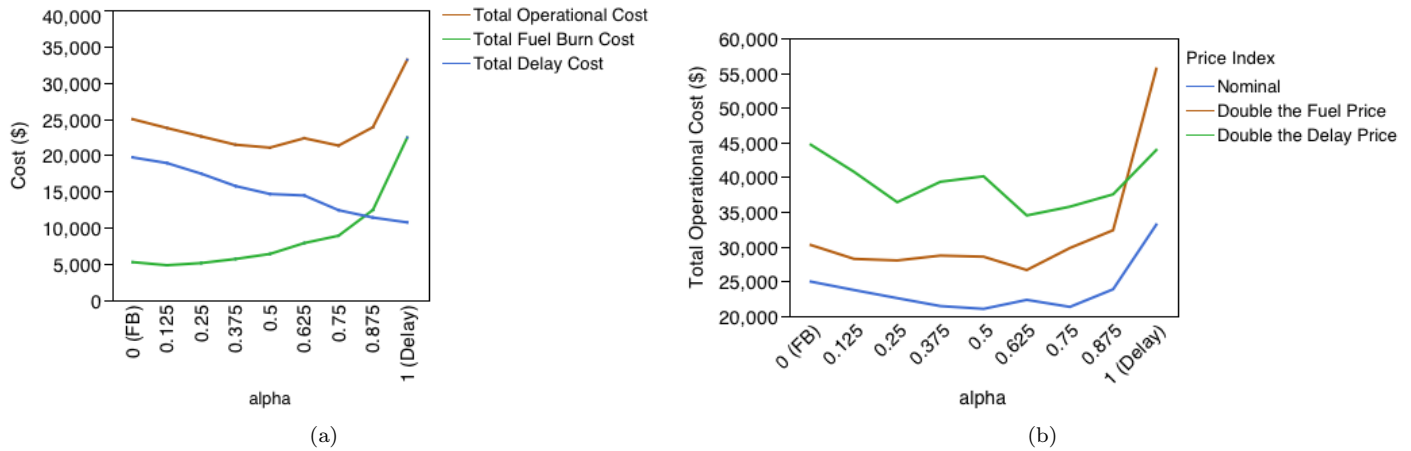


Figure 4: (a) Total Cost vs. alpha, (b) Operational Cost vs. alpha

The curve for total operational cost across alpha considering only en route conflicts is much smoother and almost symmetrical at $\alpha=0.5$. This indicates that as the importance of delay and fuel costs becomes more unbalanced, operational cost increases, and this trend is generally the same regardless which parameter (delay or fuel burn) is favored.

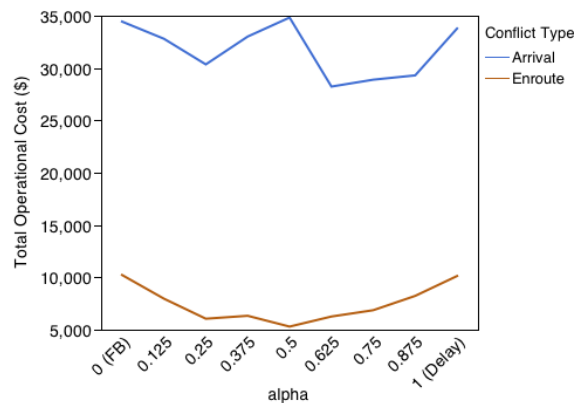


Figure 5: Total Operational Cost vs. alpha for En Route and Arrival Conflicts for Double Delay Price Index.

C. Resolution Type Costs

Further analysis illustrates the influence that the selection of resolution type has on aircraft operating costs. When resolving conflicts there are several categories of maneuvers that can be utilized to prevent a predicted conflict. The AAC Autoresolver captures most of the different resolution types used in the field, and these types are illustrated in Figure 1. For most resolution types their impact on the performance of delay and fuel burn can be generally hypothesized through intuition into the physics of the maneuvers. For instance, maneuvers such as a Direct-to which identify wind favorable shortcuts along the planned route are known to save time and fuel, step altitude climbs generally reduce fuel burn, step altitude descents generally increase fuel burn, path stretches are known to increase delay and fuel burn, and speed reductions save fuel, but increase delay. The performance of delay and fuel burn does not have a direct correlation based on resolution type, however the operational cost of resolutions creates a normalization of the two metrics and comparisons can be made. In this section, the operational cost among 13 different resolution types are investigated in an attempt to uncover which resolutions theoretically cost more than others.

Figure 6(a) provides the average operational cost per resolution executed for each resolution type. The data was computed using simulation results of $\alpha=0.5$, in order to evenly balance the cost of delay and fuel

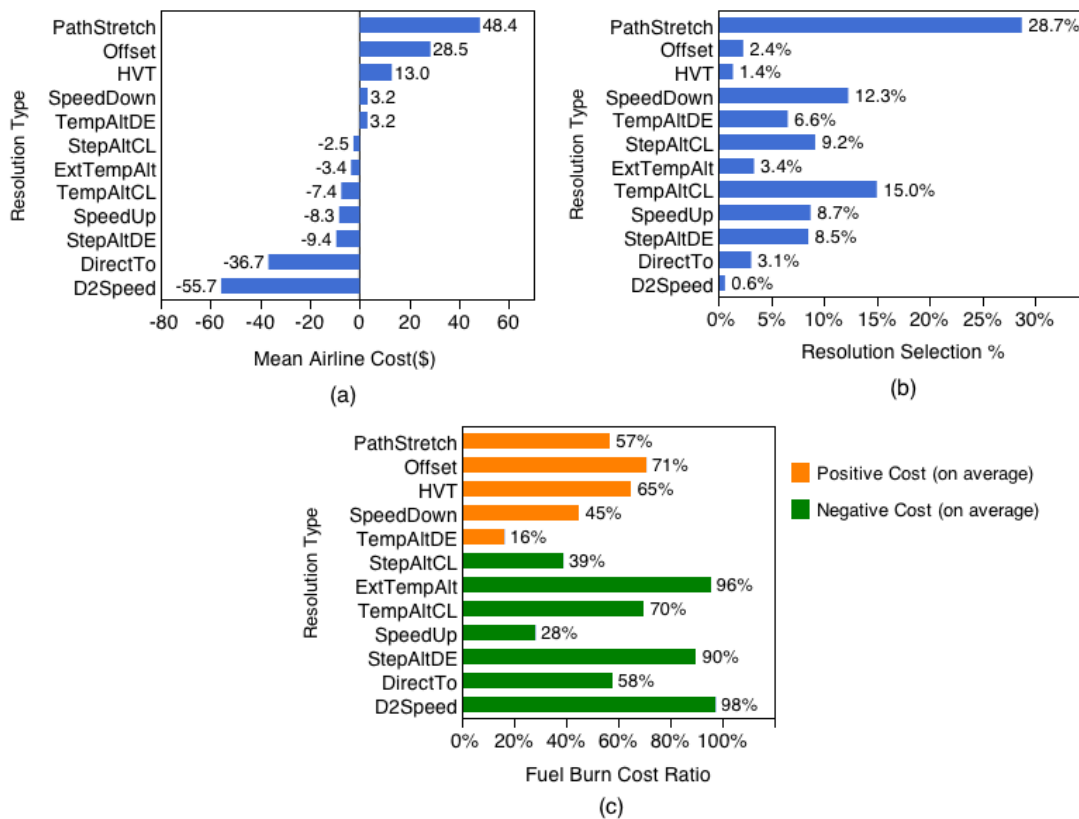


Figure 6: Comparison of results by distinct resolution types for $\alpha=0.5$ and nominal price. (a) mean operational cost per resolution type, (b) percentage of all conflicts each resolution type was selected, (c) fuel burn price ratio.

burn, and the nominal price index. As expected, the two best resolutions with respect to operational cost are Direct-to and Variable Speed Direct-to (\$36.70 and \$55.70 savings, respectively). Both maneuvers result in a shorter horizontal path thus saving fuel and delay, and Variable-Speed Direct-to (*D2Speed*) initiates a speed reduction simultaneously with a Direct-to for added benefit. The most expensive resolution types are the horizontal maneuvers: path stretch, offset, and horizontal vector turn (*HVT*). Path stretches produce the overall highest price with \$48.40 per resolution. Interestingly, seven of the 13 resolution types produce a mean price savings (negative cost), however it should be noted this does not translate to a total price savings as the magnitude of horizontal maneuvers and speed reductions overtake any savings for an incurred cost. In the speed domain, increases tend to save \$8.30 on average and reductions incur a price of \$3.20 on average. Both step altitude descents and climbs produce a mean price savings with descents saving approximately four times more than climbs. Furthermore, temporary altitude descents incur a mean price of \$3.20, by contrast temporary altitude climbs save \$7.40. A maneuver not illustrated in Figure 6 called an extended temporary altitude (*ExtTempAlt*), is a maneuver where in order to resolve a conflict an aircraft, already performing a temporary altitude, remains at the current temporary altitude for a specified period of time (i.e., 12 minutes). These maneuvers produce a mean price savings of \$3.40.

The selection rate of each resolution type is important when making comparisons, especially when a broad distribution exists as shown in Figure 6(b). In our simulations, path stretches were the single most utilized maneuver when resolving conflicts. These maneuvers are the most frequently selected (approximately 29% of the time), because they are extremely successful at creating the separation minima required to clear a conflict. Consequently, they are also the most expensive resolution, and the dominant factor in the overall price of resolving conflicts. By contrast, the Direct-to maneuver significantly reduces operational cost when selected, however is only utilized 3.1% of the time. Regardless of the optimization problem or advancements to conflict resolution algorithms, the performance of resolutions will never be more important than the

criticality of avoiding actual losses of separation. It is likely there will always be a net price to resolving conflicts.

In order to uncover which cost, delay or fuel burn, has a greater effect on the results in Figure 6(a), the percentage that derives from the price of fuel is shown in Figure 6(c). For example, the value of 57% for the path stretches means 57% of the \$48.40 per resolution comes from the price of fuel, thus 43% of the price derives from additional delay. Eight of the 13 resolution types produce mean costs mostly made up by the price of fuel burn, i.e. fuel burn price percentages greater than 50%. Most of the price savings for Variable-Speed Direct-to maneuvers come from fuel burn (98%). This is expected because the maneuver employs a speed reduction to produce zero delay when performing a Direct-to, thus no delay change to benefit from. The price of path stretches and Direct-to maneuvers are nearly impacted evenly between delay and fuel burn. Moreover, temporary altitude descents and speed increases were primarily impacted by their respective delay price (savings).

VI. Conclusions

Fast-time simulations of current-day traffic levels in two regional airspaces under nominal weather conditions were simulated to evaluate the benefit of modifying a conflict resolution algorithm to select resolution maneuvers based on minimum cost. The study employed the use of a parameter, alpha, to represent the relative importance of fuel price to the cost of delay, similar to FMS cost index. In terms of operational cost, the most efficient choice of alpha is roughly 0.5, i.e., when the cost of fuel and delay are weighted evenly. The operational cost is highest when the cost of fuel is ignored by the algorithm (i.e., $\alpha=1$), which is the case for most conflict resolution algorithms in the literature, including the baseline prototype (AAC Autoresolver) that was modified for this study.

The most cost-effective resolution maneuvers were the Direct-to and the Variable-Speed Direct-to, owing to the fact that both maneuvers result in a shorter horizontal path, thus saving time and fuel. Conversely, the most costly maneuver type was the Path Stretch. The cost of fuel burn is the predominant factor in the total operational cost of most (eight) of the 13 resolution maneuver types.

VII. Future Work

An approach for evaluating cost based conflict resolution was presented. The context for the evaluation was an approximation of delay and fuel cost. However, the price of delay varies based on the number of passengers per aircraft. In the future, an investigation of how different aircraft fleet mixes (low, medium, and high-occupancy aircraft) affect the results of selecting resolutions based on minimum cost could be performed. Furthermore, other ARTCCs could be simulated to test the results of different conflict geometries (i.e. different resolution type distribution). Each simulation in this study used the same value of alpha for every, conflict therefore, each conflict had the same defined importance of delay vs. fuel burn. Further research into calculating an optimal alpha value for each conflict based on characteristics of the flight pair could improve the operational practicality of approach. For example, a flight approaching its arrival fix may tend to prefer to minimize delay (higher alpha value), especially if it is already behind schedule and there are no apparent arrival sequencing issues. By contrast, a flight that is early may tend to prefer a fuel-efficient resolution (lower alpha value), because any further timesaving may be nullified by additional delays in the terminal.

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