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On Multiple-Layered Vortices

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June 2011

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NOMENCLATURE

E	$\nu/\Omega L^2$ = Ekman number
ε	$U/\Omega L$ = Rossby number
g	acceleration of gravity, ft/s^2 (m/s^2)
L	characteristic length of flow field, ft (m)
μ	viscosity, lb/ft/sec/g (kg/m sec)
ν	μ/ρ = kinematic viscosity, ft/s (m/s)
Ω	rotation rate of the fluid, radians/s
r	radius from center of rotation, ft (m)
ρ	density of fluid, lb/ft^3/g (kg/m^3)
Γ	$2\pi r v_\theta$, ft^2/s (m^2/s)
u_r, v_θ, w_z	velocity components in radial, circumferential, and axial directions, ft/s (m/s)
U	characteristic velocity of flow field, ft/s (m/s)
v_θ	swirl velocity, ft/s (m/s)

Subscripts

cb	cloud bottom, ft (m)

ON MULTIPLE-LAYERED VORTICES

Vernon J. Rossow[1]

Ames Research Center

SUMMARY

As part of an ongoing effort to find ways to make vortex flow fields decompose more quickly, photographs and observations are presented of vortex flow fields that indicate the presence of multiple layers of fluid rotating about a common axis. A survey of the literature indicates that multiple-layered vortices form in waterspouts, tornadoes, and lift-generated vortices of aircraft. A tentative explanation is suggested for the appearance of vortices with multiple-layered cylindrical structures. The observations and data presented are intended to improve the understanding of the formation and persistence of vortex flow fields.

I. INTRODUCTION

The structure of buoyancy- and lift-generated vortices is of scientific interest because these events often have energetic flow fields that are sometimes useful but more often pose a serious hazard to their surroundings. For safety reasons, it is reasoned that efforts should be carried out to find methods that will cause vortices either to decompose more quickly or to not form. In support of such a study, photographs and observations are presented of vortex structures that occur in laboratory experiments, in the vicinity of severe local storms, and behind aircraft in flight. The structure of vortex flow fields in general is of interest, but the feature of most interest here is a multiple-layered structure that sometimes appears in vortices as several cylindrical flow fields with a common axis of rotation. It is not known for certain whether multiple layers are always present and visualized in certain cases only by means of flow-field markers, or if multiple layers rarely occur and are visualized when present.

This paper was motivated by photographs taken of an intense waterspout[2] that indicated that the vortex structure appeared to be composed of multiple cylindrical layers of water droplets and air that were either moving upward or downward while rotating about a single axis of rotation (ref. 1). The purpose of this paper is to provide observations and photographs of multiple-layered

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[2] A waterspout is defined as a swirling body of air like a tornado or dust devil, but the vortex forms over a body of water; i.e., usually shallow coastal waters that warm readily during summer months (ref. 2).

vortex structures along with descriptions of the situations that led to the observations and the photographs taken.

The papers listed at the end of the paper (refs. 1–38) contain pictures and observations of vortices in the atmosphere along with some theoretical models and analyses. The information found in the literature cited does not indicate whether or not the presence of multiple-layered flow fields in vortices is regularly a part of vortex structures or a rare event. The study reported here also did not determine whether or not multiple layers in vortices provide an opportunity for reduction of the hazard posed by intense vortex structures. Instead, it is suggested that multiple-layered structures are not brought about by a single characteristic of the flow field, but by boundary conditions on the flow field of the vortex in the atmosphere surrounding the vortex from where it contacts the ground or water surface at its lower end, up to where it spreads out or dissipates at its upper end located in the parent cloud far above the visible funnel. Specific details on which boundary conditions are necessary were not determined. It is suggested, however, that the vortex structure may have a special characteristic at or near its upper end located near or in several updraft and downdraft cells in or in the proximity of the parent cloud. That is, the up-and-down draft cells at or near the upper end of the organized vortex (that are hidden by the parent cloud) may play a more important role in the structure of the event than the interaction of the vortex with the ground or water surface at the lower end of the vortex. It is suggested that laboratory experiments be used to define the boundary conditions that bring about multiple layers in vortices.

In order to facilitate the discussion in this paper, two appendices appear at the end of the text. Appendix A presents an overview of several of the earliest papers on the characteristics of rapidly rotating flow fields by using theoretical and experimental studies that were conducted under controlled conditions in a laboratory (refs. 3–7). The flow-field models described there illustrate the flow-field characteristics of solid-body rotation in a closed container and the rigidity that can be achieved in rapidly rotating fluids. The work is classic and probably applies to the visible parts of atmospheric vortices that are rotating rapidly as a cylindrical configuration and where fluid also appears to be moving as rapidly rotating axially symmetric cylindrical columns. The research results in these studies and the equation for the conservation of mass for axially symmetric flow fields are used in appendix A to form a basis but not a proof for the existence of multiple-layered structures in vortices.

Appendix B presents a simple relationship for an estimate of the swirl velocity at the visible surface of waterspouts (refs. 1, 19, and 38). The relationships derived then provide data for equations that make it possible to estimate the swirl velocity along the outer surface of the visible (i.e., condensation) parts of vortices. An estimate of the swirl velocity and the diameter of the vortex may then be used to estimate the magnitude of parameters that characterize the structure of and body forces within vortices.

The main part of the text of the paper begins with a section that discusses the laboratory vortex produced by Turner (ref. 8) because it is the best example found of a vortex structure with clearly defined coaxial multiple layers. Turner's experiment is of importance for the discussions that follow because it clearly shows that vortices with multiple layers can be produced with laboratory equipment for detailed and prolonged study that is not available with events that occur occasionally and unexpectedly like atmospheric vortices. The next section is devoted to an overview of the literature on the observations and study of buoyancy-driven vortices. The last technical section of the text presents numerous pictures of waterspouts and the lower parts of their parent clouds to illustrate the structure of multiple-layered vortices observed over the ocean surface around Key West, Florida. The photographs illustrate the range of sizes, shapes, and intensity of waterspouts by the patterns of condensate and water-spray patterns that waterspouts generate.

The last vortex type of flow field chosen for presentation is a lift-generated vortex that illustrates the beginning or developing part of the cylindrical part of one of the two vortices in a vortex pair shed by an aircraft (refs. 36–38). It is noted that both the initiation and the downstream or outlet end of lift-generated vortices are also quite well understood (ref. 38), but still resist accurate prediction of the entire event because the dynamics of the decomposition process of the vortex pair usually depends on the details of the structure of the atmosphere, which changes continuously. The sequence of photographs shown illustrates the roll-up of the vortex sheet shed by the lifting wing into a pair of vortices. In particular, the pictures show that the structure of one of the vortices in the pair is also composed of cylindrical layers of flow-field markers around a center.

II. MULTIPLE-LAYERED VORTEX GENERATED BY TURNER

Because the laboratory example presented by Turner (ref. 8) presents the clearest example found of a multilayered vortex, it is reproduced in figures 1 and 2 along with the titles provided by Turner. The container used in the experiments is stationary and does not appear to restrain fluid motions at the top of the fluid. As a result, viscous boundary layers form on the bottom and sides of the container, and not at the top. The picture in figure 1 clearly indicates the core region and a region or layer of rotating fluid outside of the core, indicating that fluid motion is downward in the center part of the vortex. Also indicated in the figure are several appendages of dye that might be associated with another layer further from the center of rotation of the fluid, or that may be in the process of formation of a cylindrical surface. The flow field presented in Turner's second photograph, figure 2, clearly indicates multiple layers in the upper part of the rotating flow field. It is noted that the multiple outer layers are indicated only near the top of the container and not at the center or bottom of the event. The sharply defined thin layers of dye exhibited in figure 2 indicate that the interfaces between layers of rotating fluid have a low level of mixing as if the flow is laminar. Slow diffusion of dye in a rotating fluid is expected (ref. 9).

If the fluid in Turner's apparatus were in a steady state of motion, the vertical edges of dye could be interpreted as examples of Taylor–Proudman columns. However, because the fluid is rotating inside of a container at a circumferential velocity that varies with radius, the ink and its striations indicate a flow field with cylinders of fluid that are tied together like the smaller columns of fluid above and below a sphere that were studied by Taylor and Proudman (refs. 3–6 and appendix A). The flow field illustrated in figures 1 and 2 then appears to be different and to contain more complex cylindrical forms of the columns studied by Taylor and Proudman.

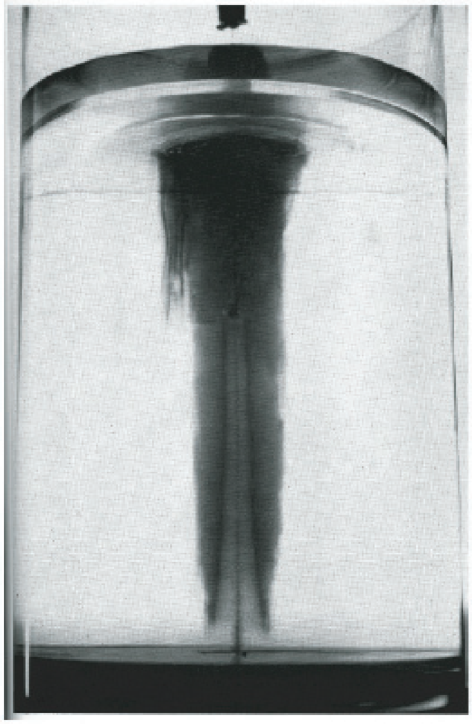


Figure 1

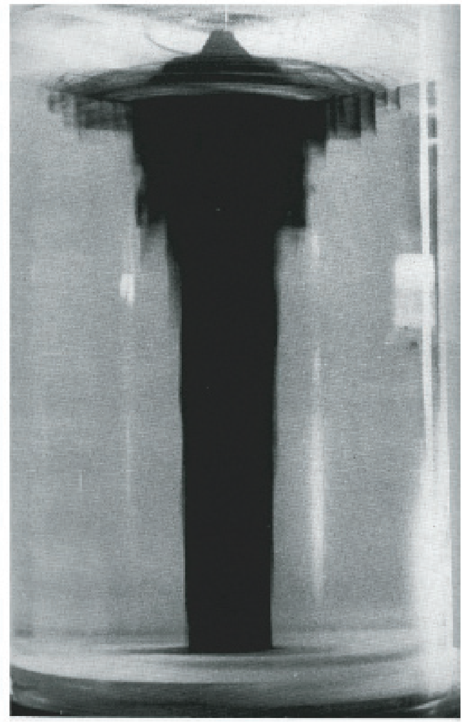


Figure 2

Figure 1. Photograph of a vortex driven from above by convection produced by a stream of air bubbles. A dye crystal on the bottom shows the upward motion in the center, and dye put in at the top marks an annular down flow. Plate 1 of Turner (ref. 8), © 1966 Cambridge University Press; reprinted with permission of Cambridge University Press.

Figure 2. Photograph of a vortex driven similarly to that in figure 1, but with deeper dye at the top and left for a longer time. The dye, and the downward motion, is confined to a cylindrical region around the axis, on reaching the bottom, fluid flows rapidly inwards in a thin boundary layer and up the center. Plate 2 of Turner (ref. 8), © 1966 Cambridge University Press; reprinted with permission of Cambridge University Press.

The theoretical model introduced in Turner's paper is devoted to finding a theoretical model for his experiments and developing a theoretical model for the core and first radial layer of rotating fluid in the vortex. Attention is not drawn to the more complex cylindrically shapes of thin ink dye markings featured in the upper one-third of the laboratory flow field shown in figure 2, nor is an explanation provided for the strange shapes of ink dye formations. It may be that multiple layers need time to develop or that it takes time for the ink dye to move into locations that mark the entire surface of the axial motion within each of the various cylindrical layers of fluid. It is important to note that Turner's experiment demonstrates that multiple layered vortices can be produced and studied with laboratory devices that can be used to model the similar larger events seen in the atmosphere. There is no indication as to whether the circumferential velocity distribution is continuous or discontinuous across the various layers.

III. OVERVIEW OF LITERATURE ON MULTIPLE-LAYERED VORTICES

A search of the literature for records of atmospheric vortices that have a multiple-layered structure found that vortices with one or two cylindrical columns at or near the center of the vortex are observed regularly (refs. 1, 2, 7–9, 16, and 27–34). A downward motion of fluid along the centerline often appears in the central core region along with a strong secondary or outer core moving in the upward direction. It is usually suggested that the downward motion along the center of the vortex is brought about by the low pressure induced there by the centrifugal loads within the layer of rotating fluid just outside of the core region. The change in vertical velocity at the ground or water surface forces the downward flow at the center of a vortex to join the upwardly moving cylinder of rotating and buoyant fluid just outside of the core region. At the radius on the ground or water surface where the downward moving fluid changes direction from downward to upward, an industrious scrubbing action at the surface brings about an intense conical spray of debris or water droplets. Photographs of waterspouts are presented in the next section to illustrate these inner characteristics of atmospheric vortices.

Only one photograph (not included here) was found of a tornado with a clearly indicated multilayered structure (page 88 of an article by Snow (ref. 10), photograph by David Hoadley). The photograph indicates a multiple-layered tornado with well-defined edges of rotating fluid at the upper part of the vortex just below the parent cloud, quite similar to the one produced by Turner in figures 1 and 2. The text that accompanied the photograph neither called attention to nor referenced the multiple layers shown in the photograph.

Another type of flow field sometimes observed around tornadoes of large size is an outer ring of two to six or more smaller vortices that orbit (like satellite vortices) about the primary strong vortex at the center of the flow field (refs. 23 and 29). Although these multiple-vortex flow fields are interesting, the vortex structures discussed in this paper are those that have a multiple-layered cylindrical flow field observed in a single vortex structure. Similarly, hurricanes when viewed from above are well known to have cloud distributions that have a spiral form around the center or eye. The spiral-shaped cloud formations might be a part of a large-scale version of multiple layered vortices, but they also are assumed to not fit the multiple-layered model of vortex structures being considered.

Most of the other publications found on the subject of vortex structures like dust devils, waterspouts, and tornadoes assume that the vortex structures being studied consisted of a core region and a single outer layer (refs. 8–34). Several efforts to model multiple-celled vortices have been made (ref. 16), but efforts to model vortex structures like those found by Turner were not found. The existence of two inner cells within vortex structures discussed previously is commonly recognized. It was also suggested that a vortex flow field with a single upwardly moving cell be called a Burgers vortex, and one with more than one cell a Sullivan vortex.

Dust devils are not discussed here because their visible structure is usually limited to a core (usually moving downward) and a single outer layer of heated air and particulates moving upward. Multiple layers of flow-field markers being carried aloft outside of the outer rotating layer are usually not observed around dust devils (refs. 33 and 34). In summary, even though numerous experiments and analyses appear to have been carried out on vortex structures with a core and a single outer layer, none appear to have considered the possibility of multiple-layered vortices.

IV. TYPICAL WATERSPOUT STRUCTURES

A. Background

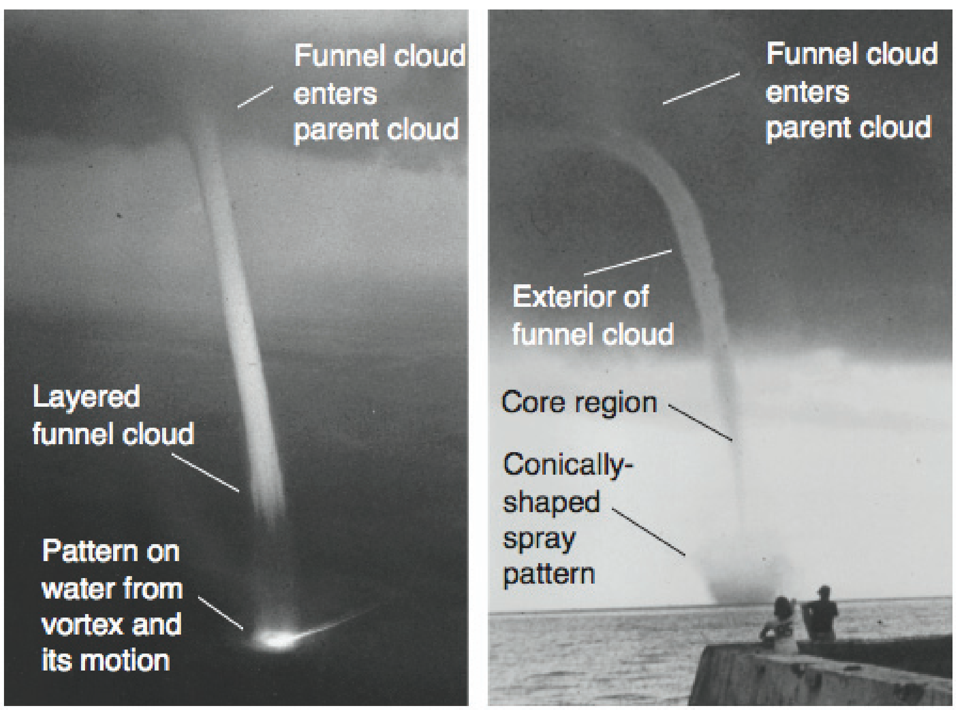
The ocean around southern Florida was chosen for study because waterspout events are common throughout the region during summer months (refs. 1 and 24–26). All atmospheric vortices are of interest to NASA because their occurrence in the vicinity of the Kennedy Space Center may affect the launch and recovery of spacecraft. The United States Navy is also interested in weather in general, and especially in severe weather events. The Key West area and the nearby Boca Chica Naval Air Station were chosen for study because it was well known that waterspouts constitute a convective vortex event that often occurs over the warm waters that surround Florida when the atmosphere is calm and low-amplitude winds are present during summer months (refs. 1, 2, and 24–25). More than 50 waterspouts were observed during a summer test period in 1968. All of the funnel clouds examined were over water. Those over or near land were not studied because of concern for damaging debris that might have been drawn aloft in the vicinity of the waterspout and its parent cloud.

The initial primary objective of the research was to determine whether there is a possible electrical connection between severe weather events like waterspouts and their parent and/or surrounding clouds. That is, is there a prominent electric field or lightning signature that could be monitored to provide a forecast of the possible occurrence of an atmospheric vortex? It was found that the parent cloud does not become electrified in any way before or while a waterspout is present. That is, no electrical indication was found for the appearance or presence of waterspouts. In fact, it was found that all of the waterspouts and their parent clouds that were studied did *not* have an electrical signature until waterspout activity had ended and rainfall began. For this reason, effort was then primarily directed at the determination of the character and occurrence of buoyancy-generated atmospheric vortices like waterspouts.

The research study that led to the photographs to be presented was set up as a cooperative program between NASA Ames Research Center and the Naval Air Station located on Boca Chica Island next to Key West, Florida. The Office of Naval Research in Washington, D.C., made the arrangements for the cooperative program. NASA Ames provided the instrumentation, the cost of its installation and removal at the end of the study, the cost of the fuel and oil used by the aircraft used in the study, and the author as observer to direct the study and to record the measurements taken. The aircraft used in the study was one of several propeller-driven Grumman S2E patrol aircraft (Wing span = 72 ft 1 in.) assigned to the Air Station. The regular assignment of the aircraft used in the investigation was to monitor ship traffic into and out of Cuba in response to the missile crisis during the 1960s. The waterspouts observations were made on a non-interference basis while the instrumented Navy aircraft was on patrol duty.

B. Photographs of Typical Waterspout Structures

The photographs presented here were all taken of waterspouts that occurred in the vicinity of Key West, Florida (e.g., ref. 1). Photographs of the parent cloud over waterspouts were not usually taken because they were so large that an isolated view and clear identification were difficult to obtain. Two typical waterspouts and the lower part of their parent cloud (i.e., where the funnel disappears from sight) are presented in figure 3. The visible surface of the funnel or cylindrically shaped vortex is water vapor brought about by the lower static pressure (and cooled air) in the swirling flow field of the vortex (appendix B). As discussed previously, the lower pressure and high swirling velocities associated with the vortex induce a column of water spray that has a conical (or funnel) or cylindrical shape that extends upward from the water surface a part of the way to the bottom of the parent cloud (fig. 3(b)). As seen in the figure, the event does not spout water but causes water from the ocean surface to be drawn aloft by the strong rotary and vertical winds at and near the water surface. Contrary to lift-generated vortex wakes that always consist of a vortex pair, waterspout vortices usually appear singly and rarely as a multiple-vortex event.



a. Waterspout seen from aircraft. b. Waterspout seen from ground.

Figure 3. Photographs of midsized waterspouts frequently observed in the vicinity of Key West, Florida. The waterspout shown in figure 3(a) is layered in the lower segment, and the waterspout shown in figure 3(b) has generated an extensive spray pattern at the water surface.

The photographs of two typical mature, modestly sized waterspouts are presented in figure 3. A core and a two-layered structure are most apparent in the lower part of the waterspout shown in figure 3(a), but they also exist in figure 3(b). A layered structure may have been present in the upper parts of the two funnels, but the photographs did not show them. The waterspout shown in figure 3(b) has generated a large conical- or bowl-shaped water-spray pattern at its base, with an intense rotary and upward velocity pattern at its lower end on the water surface. It is noted that the stepped ink dye pattern clearly shown in figure 2 of the laboratory vortex with multiple layers (ref. 8) is not observed in these photographs.

Waterspouts usually form as a single isolated vortex when a region of barely perceptible rotating air is concentrated into an intense vortex by an updraft of above-average strength in a rapidly building convective cell. It is commonly believed by waterspout watchers in the Key West region that the source of the circulation associated with a given waterspout is generated by a low-velocity wind around and/or through a rain shower. The rainfall then acts like a screen as it generates a region that contains circulation with a vertical component, which is needed for vortex generation. As indicated in the literature, waterspouts form in a region that possesses circulation and where a strong convective (updraft) cell will or has formed.

The vortex forms and persists during the earliest part of the rapid-growth stage of its parent cloud. For most waterspouts, the funnel and cylindrical shape first appear at the bottom of the parent cloud as a small-diameter translucent cylinder (fig. 4) that extends downward toward the water surface from the cloud. Sometimes, the funnel never reaches the water surface before it appears to withdraw into the bottom of the parent cloud. One of the first bits of evidence to indicate the formation of a waterspout is the presence of a region on the water surface below the funnel where the water surface has been rubbed smooth by the rotary flow field of the vortex (fig. 4). As the vortex intensifies, the “smooth rubbed surface” on the water becomes much more agitated, causing a water-spray pattern to form that indicates the direction of rotation and travel of the vortex. Thereafter, multiple layers sometimes form in the vortex even though the funnel cloud is not dense in the region near the water surface. As the waterspout ages, and perhaps intensifies, a cylindrically robust extended column of water spray is sometimes generated from the water surface up towards the bottom of the parent cloud (fig. 5).

A more intense updraft in the convective cell coupled with a large supply of air with circulation produces a more intense waterspout of longer duration. Why a waterspout appears at a given time and location is only vaguely understood. Sufficient information on the atmosphere before a waterspout event occurs would be difficult to obtain. No attempt was made during the study period to make the necessary measurements over a wide region of the ocean to determine whether and when circulation and buoyancy characteristics of the atmosphere were appropriate for the formation and intensity of waterspout events. At the time when the foregoing observational flights were being carried out, no effort was made to predict the occurrence of waterspouts because weather predictions by the Naval Air Station indicated that waterspout activity was likely almost every day during the summer while the aircraft was part of the program.

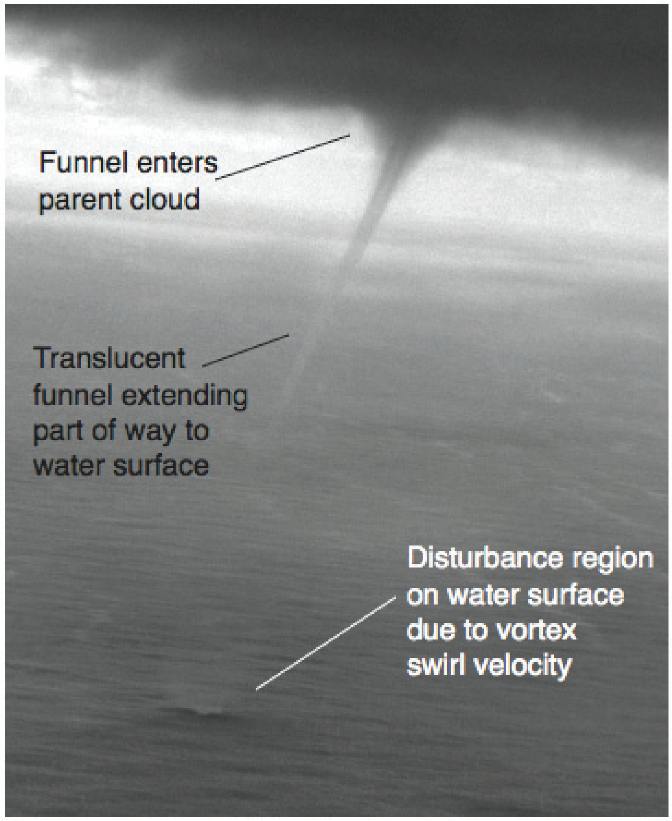


Figure 4. Early appearance of waterspout as visible funnel with translucent core that does extend to water surface, as indicated by spray and surface disturbances on the water.

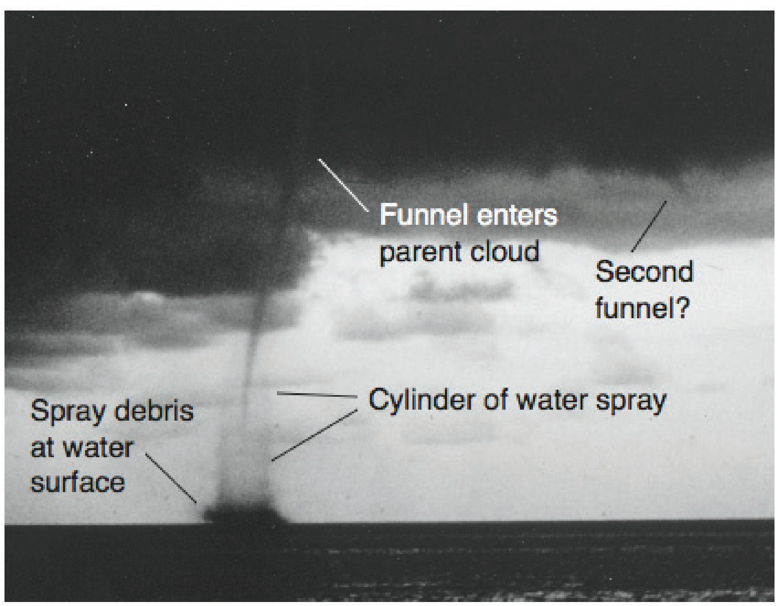


Figure 5. Waterspout with conical region of intense water spray near surface of water and a cylindrical column of water spray that extends upward from water surface several hundred ft even though vortex is not strong enough to visualize waterspout core region at lower levels of the event. Cloud bottom is at about 1000 ft (300 m). With permission by Gwin (ref. 27).

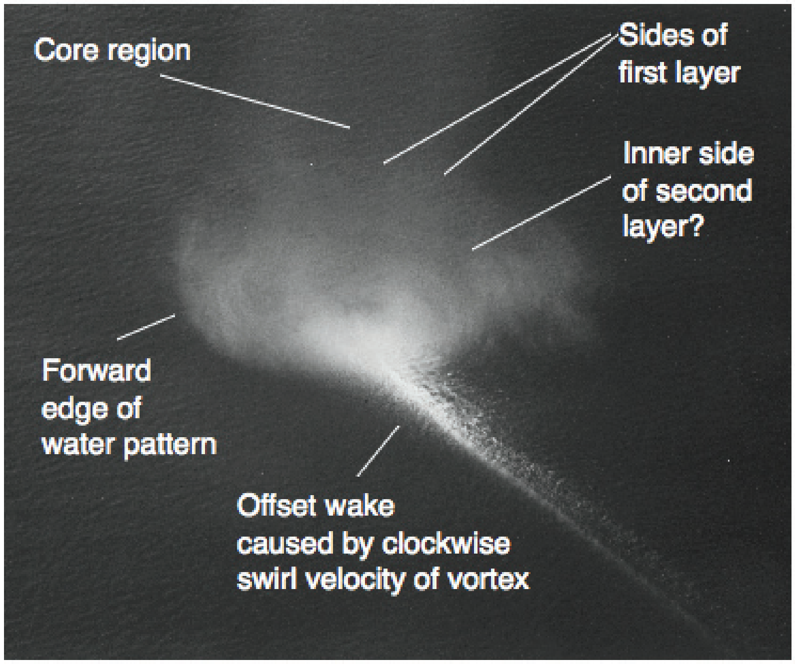


Figure 6. View from aircraft of pattern on water surface caused by clockwise swirl of vortex flow field. Funnel and core along with two (?) outer layers of vortex flow field appear to indicate at least two cylindrically layers. Remainder of visible funnel gave no indications of multiple layers. Funnel diameter was about same as wingspan of aircraft.

It was found that waterspouts rotate both clockwise and counterclockwise. An indication of the structure of the vortex/water interaction is presented as a diagram in figure 8 of the previous paper on the subject (ref. 1). A clockwise example of rotation is shown in figure 6. When a waterspout occurs in a region where rainfall is present, the waterspout disappears. It is reasoned that a waterspout disappears when the updraft driving the waterspout is disrupted by falling rain. At other times the shape of the outer surface of the visible spout becomes irregular or the ambient flow field becomes too disorganized for formation of a highly organized vortex. Sometimes the waterspout appears to quit without rainfall because the air with enough circulation to support the vortex has been depleted so that the funnel cloud withdraws into its parent cloud. In summary, waterspouts occur in a wide range of sizes from a small funnel at the base of the parent cloud to diameters that are sometimes as much as 100 m (300 ft) or more. They do not appear to be produced or to persist under mature convective cells that have lightning and rainfall.

The response of the aircraft during passage from clear air into the region under a parent cloud with a waterspout did not indicate the presence of a strong updraft. It was concluded that the updraft that supports the generation and persistence of a waterspout must either be so small that it does not affect the aircraft or so large in plan-view diameter that the aircraft never was outside of the updraft region. It is also possible that a gradual transition takes place in the vertical velocity between the region where no updraft exists and where the updraft is strong enough to support a waterspout but not abrupt enough to be detectable by the flight path of the aircraft.

Even when the aircraft was deliberately flown through a waterspout, the only perceptible response of the aircraft was in yaw and not in the vertical direction. (Note that it is the author's opinion that it is always too hazardous to fly into or through an atmospheric vortex of any size or kind or into or through a lift-generated vortex.) The passage of the aircraft through the funnel did not seem to bother the funnel and definitely did not seem to have any effect on the demise or enhancement of the waterspout. The size and weight of the aircraft were both small compared with the size of the atmosphere devoted to the vortex event.

When the waterspout is strong enough to extend from the bottom of the parent cloud to the water surface, water spray is produced within a large cylindrical or conically shaped region composed of a spray of salt water around the funnel. (To reduce the likelihood of corrosion on the test aircraft, the pilots always made certain to fly through a rain shower to wash any salt residue off of the instrumented aircraft after a flight near a waterspout.) As the parent cloud grows in height, rain begins to form in the upper part of the cloud. The rain falling from the parent cloud causes the waterspout under the cloud to quit and disappear. Instrumentation installed on the aircraft indicated that electrification of the parent cloud then begins. Lightning does not begin until the cloud has grown further, but well below the altitudes where ice crystals begin to form in the parent cloud. The observations made indicate that none of the formation or demise processes associated with waterspouts appears to be influenced by any of the electrical processes going on in the parent cloud, nor within nearby convective cells.

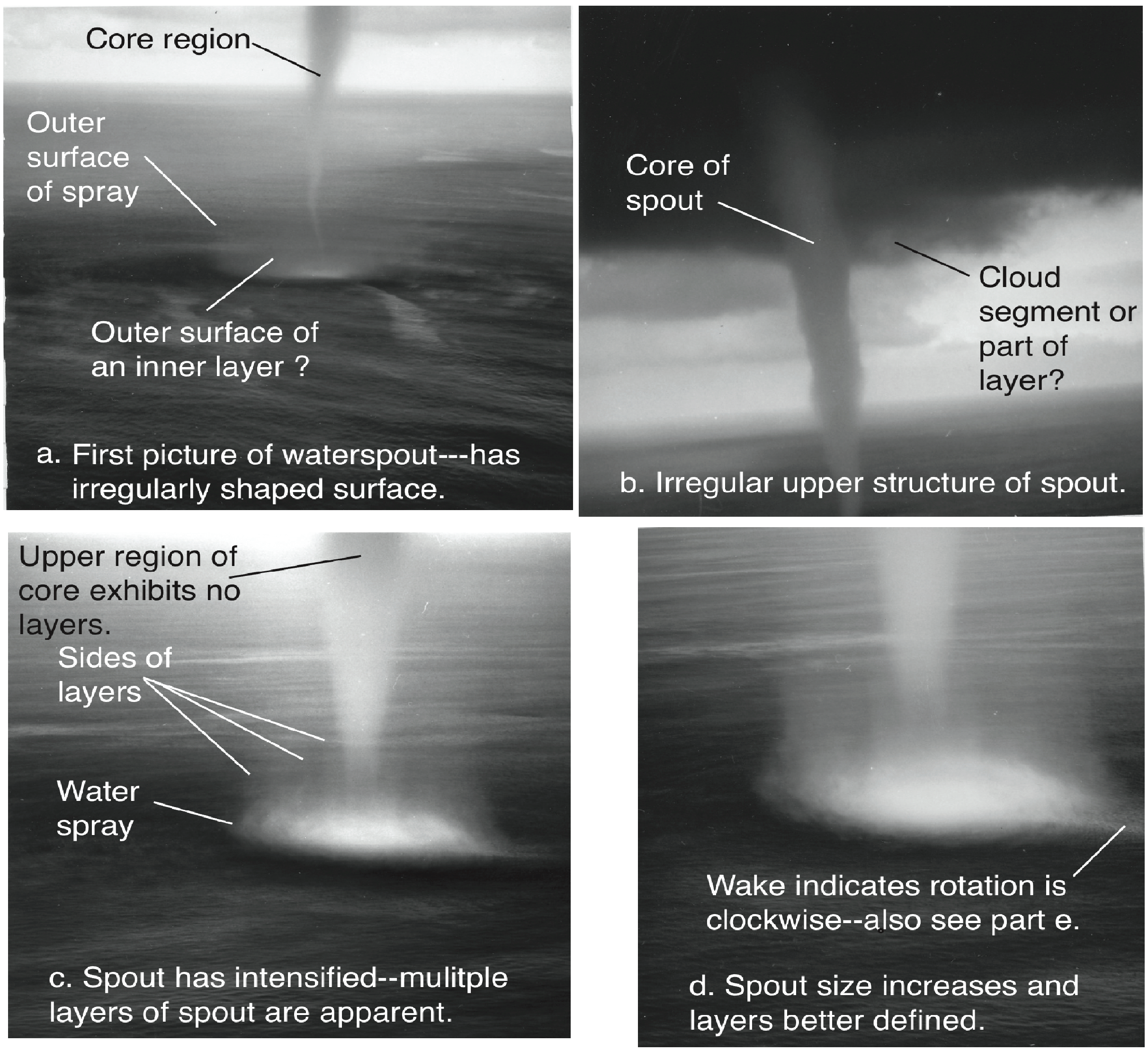
Many photographs of tornadoes (large and intense buoyancy-driven vortices over land) show the funnel cloud as an axially symmetric triangular shape (small end on the ground) rather than as a cylindrical shape that is usually observed for waterspouts. Because the tornadoes photographed usually have a large dust and debris pattern that is also associated with the funnel (refs. 16–23), well-defined multilayered cylinders of debris appear to be rare. Moderate-intensity tornadoes appear to have a funnel shape that is similar to intense waterspouts.

The intensity of the rotational velocities in a waterspout do not seem to be directly related to its size because some small waterspouts have a lot of vertical spray of water around them, whereas a large vortex may have only a small or moderate amount of spray. In other words, large-diameter waterspouts were observed that had a very small amount (height) of water spray, and waterspouts were also observed that had small-diameter funnels with a large height of water spray (fig. 5). That is, diameter of and swirl velocity in waterspout events did not seem to be related to each other.

Interestingly, the axis of the funnel during the intense part of the existence of a waterspout is usually vertical and straight. As the funnel ages, the funnel usually begins to lean sideways and then take on curvature. Shortly thereafter, the waterspout begins to decompose and disappear. The consistency of the process suggests that curvature of the funnel results in Coriolis forces that mix the fluid in the funnel that then causes its organization to decompose. The observation suggests that the best way to decompose a vortex structure is to bring about curvature as strongly as possible in the axis or centerline of vortices to cause large-scale mixing in the funnel region.

V. PICTURES OF AN INTENSE MULTILAYERED WATERSPOUT

The most intense and complex flow field observed in a waterspout gave the best indication of possible cylindrical features in their flow fields (figs. 7(a)–(d) and 7 (e)–(h)).



a. First picture of waterspout---has irregularly shaped surface.

b. Irregular upper structure of spout.

c. Spout has intensified--mulitple layers of spout are apparent.

d. Spout size increases and layers better defined.

Figure 7(a)–7(d)

Figure 7. Photographs of largest waterspout observed during study period in region around Key West, Florida. Note layered structures in condensate in, around, and along the waterspout and in the spray at water surface.

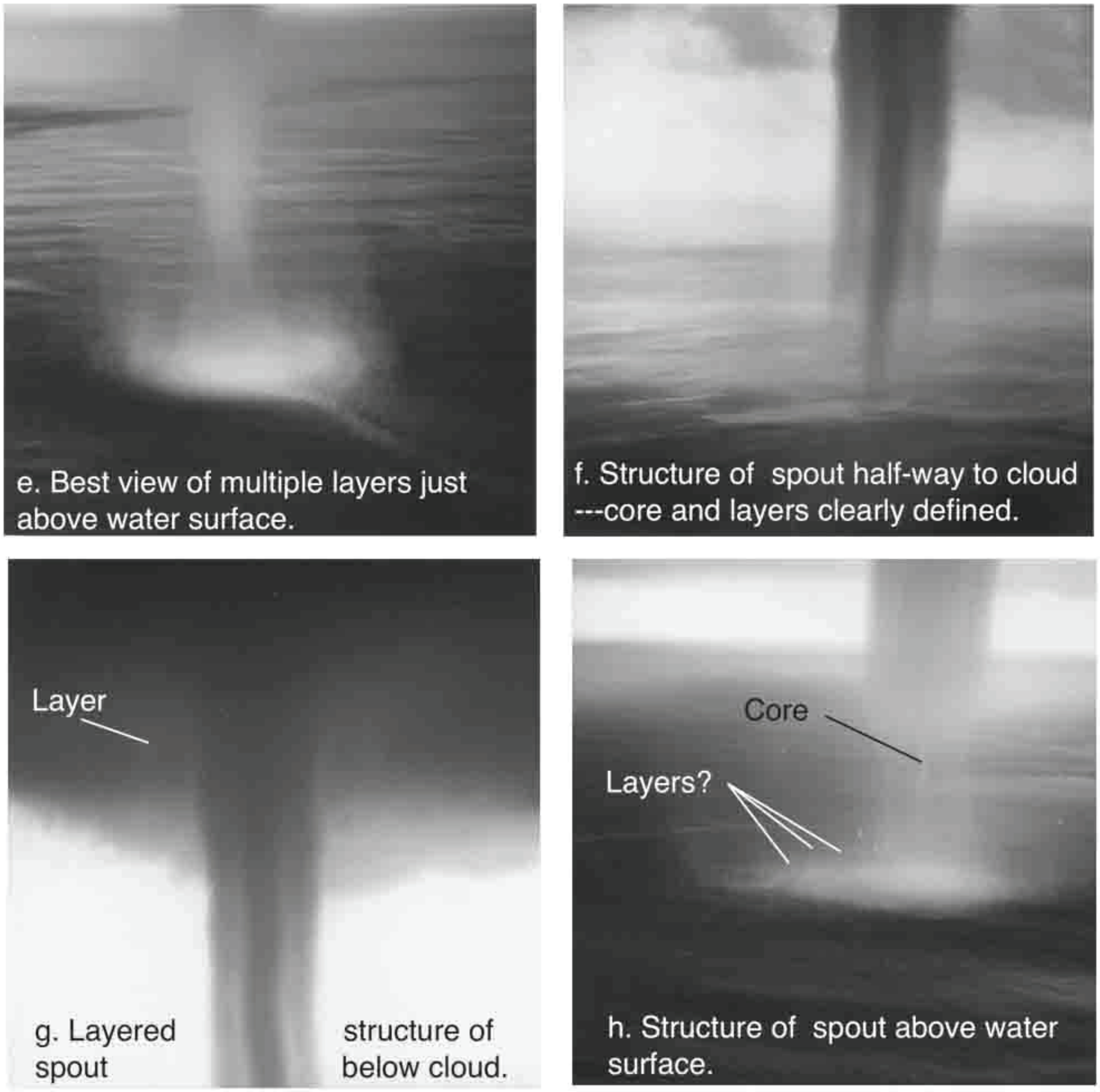


Figure 7(e)–7(h) (concluded).

The multiple-layered structure indicated in the figures by water vapor and spray at the water surface was also visible at the bottom of the parent cloud as water vapor. Leaders on the figures indicate features of interest. Unfortunately, the cylindrical layers are hard to identify because they are usually indicated only by water vapor and not by robust spray patterns. Because the pilot of the aircraft (Cmdr. P. Carr—the author was not on this patrol) recognized the size and special character of the event, numerous photographs were taken and made available for the study. It was this event that exhibited the most extensive example of multiple cylindrical layers in a waterspout during the test period.

The several regions of water vapor and spray shown in figure 7 indicate that multiple coaxial cylindrical layers are present in the vortex structure. The appearance of these narrow regions of condensate and spray at both the water surface and the cloud bottom (but not in between) indicates that the cylinders of fluid probably persisted from the water surface to the cloud bottom and probably into the parent cloud. (Also see appendix A.) Display of different cylinders of fluid motion by the changes in the texture of the flow field of the vortex indicates that various layers in the vortex are moving in up and down directions, as well as rotating. As pointed out in appendix A, such a configuration of fluid motion does not contradict the restraints placed on radial and circumferential velocity variations in rotating flow fields. In other words, vertical motion of a cylinder of fluid as a whole appears to be permissible according to theoretical restraints placed on rotating flow fields. Because the upward or downward flow in adjacent cylinders must turn around at the water surface, considerable shear and agitation of the water surface is brought about as evidenced by the large spray patterns shown in the figures. How the corresponding region of change in direction occurs at the upper end region of the vortex in the parent cloud is not known because only variations in buoyancy forces and air currents take place there and no hard surfaces are present. Because the flow of fluid along concentric cylinders also occurs in lift-generated vortices where end restraints are present only at the generating aircraft, a smoke-marked example is illustrated in the next section.

VI. MULTIPLE-LAYERED VORTEX GENERATED BY LIFTING WING

A. Background

A sequence of photographs of the motion of fluid within a lift-generated vortex is now presented to illustrate the fact that multiple-layered vortices also exist in circumstances that do not have an end constraint (like a ground or water surface) on the flow field of the vortex being observed, in contrast with the situations associated with waterspouts and tornadoes. The example chosen is a layered-vortex structure that was produced at the Wake Vortex Test Facility of the National Oceanic and Atmospheric Administration (NOAA) research facility near Idaho Falls, Idaho (refs. 35–38). Introduction of smoke into lift-generated vortices was part of a cooperative research effort between the NOAA station and the NASA Wake Vortex Alleviation Program of the late 1960s and 1970s. The example illustrates the first part of the time-dependent flow field within the port vortex of the pair that trails from the lifting wing on a McDonnell Douglas DC-3 Dakota aircraft (two-engine propeller-driven aircraft with wingspan of 95 ft). The pictures were taken near the beginning of the NASA Wake Vortex Alleviation Research Program in order to produce a training film for pilots and controllers on the hazards posed by lift-generated vortex wakes of aircraft (ref. 38). It was the intent of the display to indicate the size and intensity of the well-organized structure of the swirling velocities around the center of a lift-generated vortex to produce a hazardous region along the flight path of the wake-generating aircraft.

The purpose of the pictures shown in figure 8 is to call attention to the fact that the cylindrically layered characteristic found in waterspouts is also present in lift-generated vortices. The different axial velocities present at different radii in lift-generated vortices is believed to come about because the flow field at one radius contains air from the boundary layer on the surface of the aircraft causing the flow at that radius to be toward the generating aircraft. At the other extreme, the flow field at another radius, which includes fluid from the propulsion system, causes another layer of fluid in the vortex to travel away from the generating aircraft. Layers of fluid at other radii in the trailing vortices also move relative to the ground plane either toward or away from the generating aircraft at various velocities, depending on where the fluid had its origin. At larger radii the flow in cylindrical layers is nearly stationary relative to the ground when the wind velocity is negligible.

The film and pictures were produced to illustrate the structure of the swirling part of the flow field (and its hazardous character) shed from the two sides of an aircraft wing. The hazard consists of an overpowering rolling moment induced on the wing of any following aircraft that penetrates either of the two vortices in the pair. Lift-generated vortices have been studied extensively because they are hazardous only at certain times and not at others (ref. 38). For example, when a large aircraft penetrates a vortex shed by a smaller aircraft, the hazard is small. However, if a small aircraft encounters a vortex generated by a larger aircraft, the vortex-induced rolling moment might overpower the roll-control authority of the ailerons on board the smaller aircraft. In addition, it has been found experimentally that the swirling structure of vortices tends to persist much longer than expected when small-scale, or boundary layer types, of turbulence exist than when large-scale atmospheric types of turbulence are present. The persistence characteristic of vortices and the hazard they pose limit the capacity of airports more than any other feature of safe air traffic management in the vicinity of airports (ref. 38).

A film intended to illustrate the swirling character and hazard posed by lift-generated vortex pairs was used as the first part of a training film for pilots and controllers. The film is narrated by Robert Jacobsen, a NASA engineer assigned to the project. The vortex shown in the film was generated by a DC-3 aircraft. The motions of air inside of the flow field were marked by using a single smoke generator at the top of a weather tower to show how smoke-marked atmospheric streamers wrap around the center of the vortex shed by the left wing of the aircraft. The illustration used here is the first of many produced by using instrumented towers. Similar smoke-marked scenes were produced for many years with increasing sophistication, such as multicolored smoke streamers and hot-wire wind anemometers (refs. 35–38). The simplicity of the example shown made it ideal for presentation as a wing-generated vortex with a multiple-layered radial structure.

B. Pictures of Multiple-Layered Lift-Generated Vortex

The 12 pictures presented in figure 8 illustrate how the smoke emitted from a source located near the top of a meteorological tower is drawn as a function of time into the port or left-side lift-generated vortex shed by a DC-3 aircraft (Wingspan = 95 ft). The aircraft flew from left to right just before the pictures shown in figure 8 were taken. The smoke streamer shown in figure 8 first indicates that the ambient wind contains low-intensity turbulence eddies. Despite the presence of these background disturbances in the atmosphere, the swirling motion in the vortex is strong enough to dominate not only the turbulence in the ambient wind but also any eddies shed by the aircraft surfaces and by the propellers used to propel the aircraft. As a consequence, a large center portion of the swirling motion of the vortex consists of cylindrical layers that appear to be laminar.

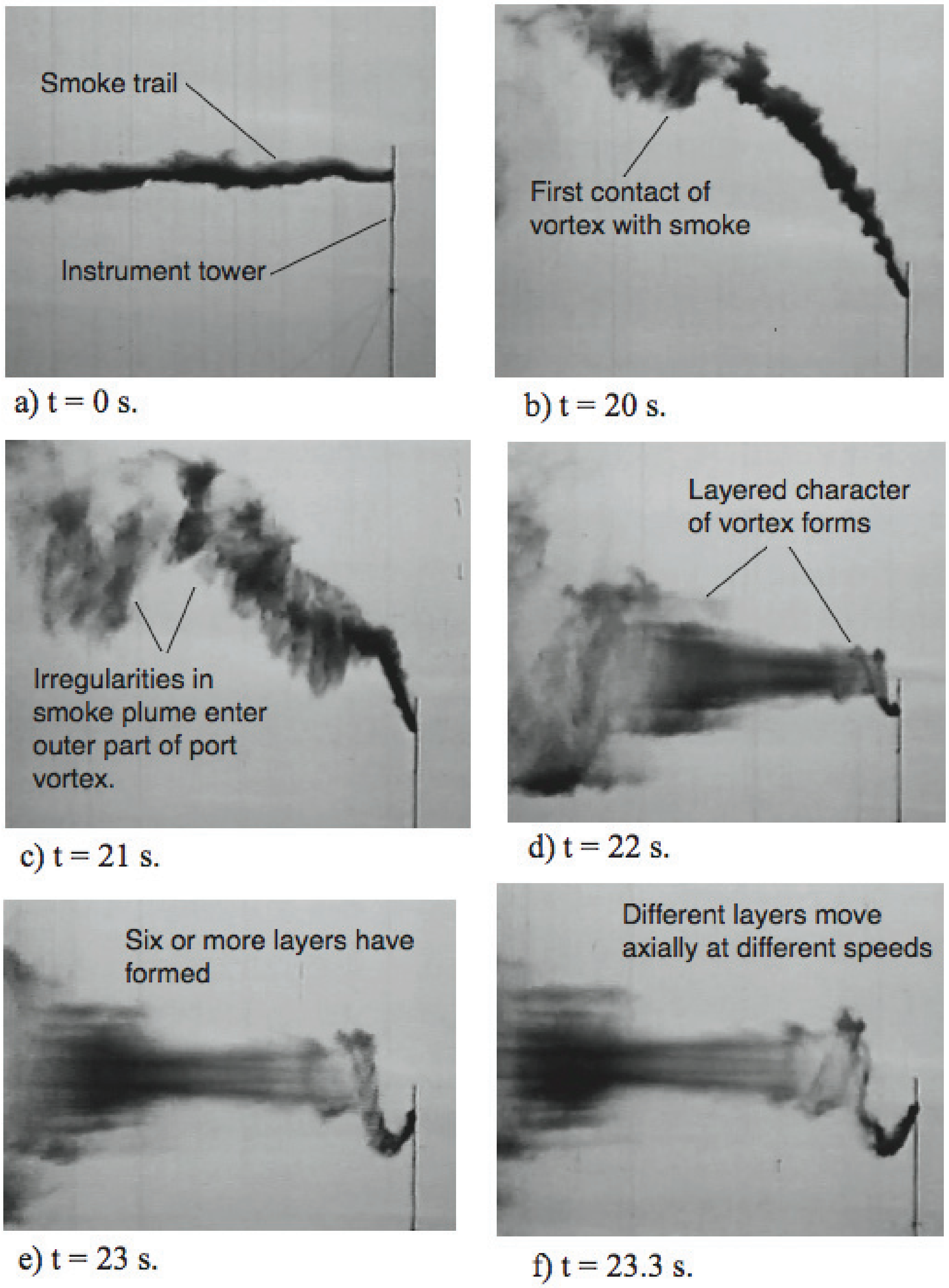


Figure 8. Smoke from meteorological tower used to visualize rollup of lift-generated vortex wake shed by DC-3 aircraft. Pictures display envelopment of smoke into layered structure of port (left) vortex as a function of time.

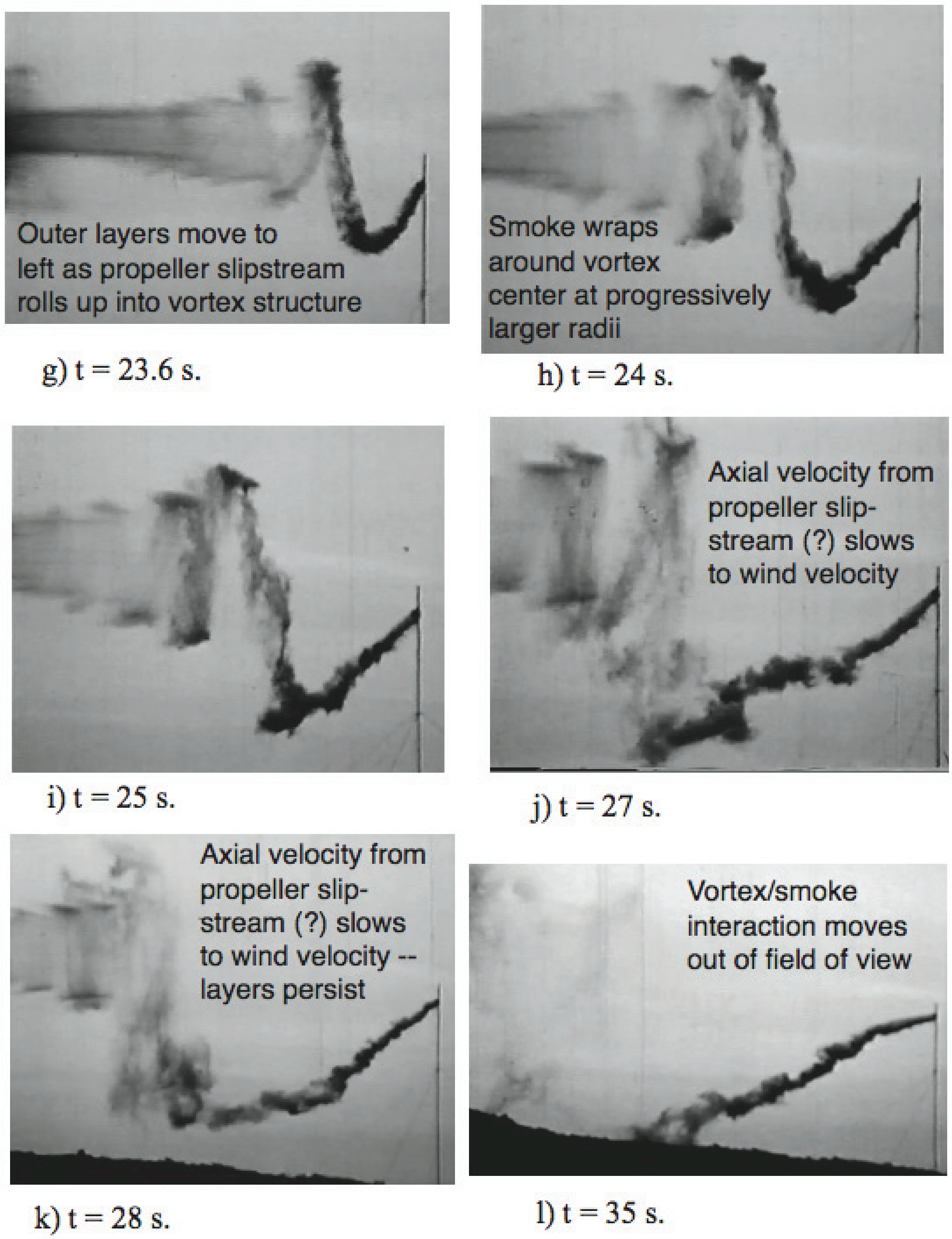


Figure 8 (concluded).

When these flights were observed from the ground, the various cylindrical smoke layers were found to move relative to each other either in a direction toward or away from the wake-generating aircraft. The direction of travel of smoke segments depended on whether the air involved was part of the boundary layer on the surface of the aircraft or part of the slipstream of the propeller. Such a process indicates that the fluid in the various layers was drawn from different parts of the aircraft; e.g., from the boundary layer on the wing or from the slipstream of the propeller in the present case. It is also quite possible that the magnitude of the swirl velocity within each layer has a different energy level even though it is in the same swirl direction. Therefore, the display of concentric cylindrical streamlines in a vortex structure is an indication that the flow field has divided into separate cylindrical flow regions and that it is close to laminar. For these reasons, the flow field, and the hazard it poses, will tend to persist longer than if the flow field were turbulent. These observations prompted observers to describe the flow field of vortices as one that had the capability to make a turbulent flow field laminar, or to "laminarize" the flow field where the vortex is located.

Similar flow-field structures were especially apparent in the wakes of the B-747 when the aircraft carried out low-level (below 500 ft) fly-bys with four wing-mounted generators of vaporized mineral oil to mark the vortex-wake structures. The wake structures shed by several different span loadings on the wake-generating wing (refs. 35–38) clearly indicate a variety of vortex cylinders shed by the aircraft that changed as the span loading on the aircraft wing was modified. The flow-field visualization material clearly indicates a variety of different cylindrical or tubular vortex structures or layers in the flow field that were clearly and sharply defined until their axes became sinuous. As with the wake of the DC-3, the cylinders or layers at different radii and with the same center of rotation often move in different directions along their common axis. That is, at one radius the cylinder moved toward the generating aircraft (as if it were made up of boundary layer air trailed by the aircraft), and another cylindrical layer moved away from the generating aircraft (as if it were made up of exhaust gases from the engines). Even small amounts of curvature appear to cause the tubular and layered vortex structures to mix and quickly disappear. The vortices then become large turbulent-looking vortex structures that look like a group of eddies rather than a boundary layer type of random small-scale turbulence. Thereafter, identifiable vortex structures were no longer clearly defined (refs. 35–38).

There was a time in the research program when it was believed that layered structures observed in the vortex wakes of aircraft were considered as being due to centrifuge of condensate droplets from one radius to another. Further consideration and the foregoing observations indicate that such an explanation for layered structures is not viable because the multiple layers observed cannot be formed by processes like centrifugal separation of particles.

VII. CONCLUSIONS

The observations and photographs presented illustrate a cylindrically layered characteristic in both buoyancy- and lift-generated vortices. Each cylinder of fluid appears to rotate and to have different finite velocities parallel to the axis of the vortex. From the information available, it could not be determined whether the cylindrical layers all had the same size and velocity, but continuity of flow indicates that the axial fluid velocities are probably related if all of the fluid in a downward moving cylinder also moves upward in the next outer cylinder. Viscous interaction of adjacent cylinders appears to be small in the waterspouts observed, but such may not be the case at altitudes large enough to be hidden by the parent cloud. The information presented indicates that a layered feature is common to many vortex structures and may be produced by several different processes.

The information presented does not indicate whether the presence of multiple layers of fluid in the flow field of vortex structures accelerates or prolongs the demise of the event. Therefore, the information presented does not indicate whether vortices behave much differently whether they have or do not have a multiple-layered structure. It is commonly accepted that regions of laminar flow in vortices usually encourage their persistence. It is fortunate that the layered characteristic observed in atmospheric vortices and in the lift-generated vortices that trail behind aircraft can be produced in the laboratory. It is then feasible to simulate these structures at smaller scale under controlled conditions to perhaps answer some of the questions regarding the structure and persistence of multiple-layered vortices.

APPENDIX A:

FLUID DYNAMICS OF RAPIDLY ROTATING FLOW FIELDS

Research efforts by Proudman (ref. 3), Taylor (refs. 4–6), and later expanded upon by Greenspan (ref. 7) appear to have application to the dynamics of the intense visible parts of the structure of vortex events. Part of their research was devoted to the study of the formation of relatively rigid cylindrical columns of fluid above and below a sphere embedded in fluid that is rapidly rotating in a circular container with closed ends. It was found that the radial or circumferential motion of the flow of fluid around objects like a small sphere is strongly resisted so that the nature of the flow of fluid changes from a three-dimensional one to a two-dimensional one. That is, instead of a three-dimensional flow field around the sphere observed in quiescent fluids (i.e., like the one that occurs in a non-rotating fluid), the motion becomes one where the fluid above and below the sphere moves with the sphere to produce a moving column of fluid instead of the flow field around an isolated sphere. It is pointed out that the change in the flow field around the sphere from a conventional one to where the dynamics within rapidly rotating flow fields is governed by parameters like the Ekman number ($E = \nu/\Omega L^2$) and the Rossby number ($\varepsilon = U/\Omega L$). The quantity ν is the kinematic viscosity, Ω is the rotation rate of the fluid, U is a characteristic velocity, and L is a characteristic length of the flow field (ref. 7). The smaller the value of these two characteristic parameters, the greater is the influence of rotation on the rigidity of the fluid dynamics within the rotating flow field. In the experiments carried out by the foregoing researchers, the effect of rapid rotation on columnar rigidity along the axis of rotation is found to increase roughly as the square of the rotation rate, Ω. It is noted here that the parameter that governs the centrifugal force per unit volume, Ωr, increases linearly with radius when solid-body rotation is present and inversely proportional to radius when the circulation, $\Gamma = 2\pi r v_\theta$, in the fluid is constant with radius, as in potential vortices.

Another approach indicates that the rigidity of rapidly rotating flow fields may be constrained in the radial and circumferential directions but not in the flow direction that is parallel to the axis of rotation. The continuity equation, or the conservation of mass, for rotating flow fields of an incompressible fluid is given by

$$\partial u_r/\partial r + u_r/r + \partial v_\theta/r\partial\theta + \partial w_z/\partial z = 0 \tag{A1}$$

where, u_r, v_θ, and w_z represent the velocity components in the radial, circumferential, and axial directions. It is assumed that the axis of rotation is aligned with the z axis. When the flow field is axially symmetric and in rapid rotation, all derivatives of the velocity components with respect to the meridian angle, θ, vanish. The radial velocity, u_r, also vanishes because the rapidly rotating characteristic of the flow field restrains radial velocities (refs. 4–7). As a result, the equation for the conservation of mass reduces to

$$\partial w_z/\partial z = 0 \tag{A2}$$

Equation (A2) specifies that the axial velocity parallel to the axis cannot change with distance along the axis, but not that it needs to be zero. In other words, the movement of a cylindrical layer of fluid in the axial direction is permitted, and the velocity of movement of any given cylindrical layer need not be the same as any other layer, if the boundary conditions on axial flow are not violated. Equation (A2) also states that the axial velocity of any given cylinder of fluid must be constant as a function of axial distance and meridian angle until the flow field encounters an end restraint like the bottom of a laboratory container or the ground or water surface in an atmospheric vortex, or the vortex structure changes shape. Conditions at the upper end of atmospheric vortices where the vortex becomes dispersed within the parent cloud are believed to provide a range of conditions that will accommodate and might produce different along-axis velocities at different radii in the intense part of the vortex. It is for these reasons that each axially symmetric layer of fluid in the cylindrical part of the vortex must move as a unit parallel to the axis of the flow field until boundary conditions along the vortex change. When a surface is placed at the lower end of vortices, upward or downward fluid motion at one radius must then turn around and move in an opposite direction. Such changes in vertical direction are suggested in some of the photographs taken of waterspouts at the water surface by the intense spray patterns observed there.

The foregoing discussion indicates that the magnitude and direction of the axial velocity must be defined by criteria other than the vortex itself. For example, the lower end of a tornado vortex is at the surface of the Earth. Interaction of the rotating flow field with the rigid surface at the ground brings about a complicated three-dimensional flow field as the downward motion of the fluid near the centerline changes from the innermost layer to the next outer layer, which is moving upwards. Angular momentum of the fluid from the inner cylinder to the outer one becomes mixed with the outer flow field and is distorted in the turn and by interaction with the ground or water surface. If other cylindrical layers of fluid are moving parallel to the axis of the vortex, the upward- or downward-moving column must also turn and move downward or upward in an adjoining layer. Such a process is regularly observed in dust devils as the downward-moving core flow first turns into a radial direction and then into an upward-moving column outboard of the core region. Because the turn is rapid and over a short distance, wind shear is robust enough to pick up debris on the ground and throw it outboard and upward. The debris pattern is then a mixed pattern that is funnel-shaped with the large end on the top. The boundary conditions that govern the magnitude and direction of the axial flow in the vortex then depend strongly on the structure of the air in the formation region of the vortex and in the parent cloud where the vortex structure decomposes. Unfortunately, the upper end of the vortex is invisible to the naked eye and may be quite complicated and different for each tornado or waterspout event (refs. 20–22).

APPENDIX B:

ESTIMATION OF SWIRL VELOCITY AND STRENGTH OF WATERSPOUTS

The swirl velocity at the periphery of the condensate that identifies the visible surface of a waterspout or tornado funnel as a function of distance below the cloud bottom can be derived from the energy equation as (refs. 1 and 19)

$$v_\theta \approx [2g(z_{cb} - z)]^{1/2} \tag{B1}$$

where v_θ is the swirl velocity at the outer edge of the visible waterspout funnel cloud, g is the acceleration of gravity, z_{cb} is the altitude of the cloud bottom in meters (ft), and z is the altitude of the outer edge of the funnel where the swirl velocity is being estimated. For example, if the location of interest on the funnel is about 1000 ft (300 m) below the bottom of the parent cloud, the swirl velocity at that location is estimated at about 250 ft/s (76 m/s). Because the location chosen for the estimate is at the outer edge of the condensation region, equation (B1) yields the swirl velocity associated with the funnel surface at the altitude chosen for study. Outside of the funnel surface the velocity is smaller because condensation is not visible.

The circulation within the funnel can then be estimated using the definition for circulation content, written as,

$$\Gamma = 2\pi r v_\theta = d\pi v_\theta \tag{B2}$$

where v_θ is the foregoing estimated swirl velocity and d is a visual estimate of the diameter (twice the radius) of the visible funnel. The circulation estimated is assumed to have been concentrated within the visible funnel by the updraft, and does not include any circulation contained in the fluid outside of the visible funnel.

As an example, the base of the parent cloud over a waterspout in the waters around southern Florida is usually at around 1000 ft. Based on that distance as a reference, some funnel diameters are estimated at around 50 ft. If the swirl velocity estimated previously, $v_\theta \approx 250$ ft/s (76 m/s) is used for v_θ the circulation in the funnel is estimated at around $\Gamma \approx 40{,}000\ \text{ft}^2/\text{s}$. For comparison, the circulation content of the lift-generated vortex shed by a 700,000-lb aircraft with a wing span a little over 200 ft and landing at about 200 ft/s is equal to about $\Gamma \approx (Wt/\rho U_\infty)$, b' ≈ 200 ft, or 10,000 ft^2/s. In addition, the angular rotational rate is estimated as

$$\Omega = v_\theta/r \approx 250/25 = 10 \text{ radians/s} \tag{B3}$$

The foregoing rough estimate indicates that the circulation content or strength of a waterspout of modest size is about four times as large as one shed by a large aircraft during landing operations. The centrifugal acceleration of an element of fluid at the visible surface of the vortex is estimated as

$$a = \Omega^2 r \approx 2500 \text{ ft/s}^2 \approx 78 \text{ g's} \tag{B4}$$

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