

# A Final Approach Trajectory Model for Current Operations

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Predicting accurate trajectories with limited intent information is a challenge faced by air traffic management decision support tools in operation today. One such tool is the FAA's Terminal Proximity Alert system which is intended to assist controllers in maintaining safe separation of arrival aircraft during final approach. In an effort to improve the performance of such tools, two final approach trajectory models are proposed; one based on polynomial interpolation, the other on the Fourier transform. These models were tested against actual traffic data and used to study effects of the key final approach trajectory modeling parameters of wind, aircraft type, and weight class, on trajectory prediction accuracy. Using only the limited intent data available to today's ATM system, both the polynomial interpolation and Fourier transform models showed improved trajectory prediction accuracy over a baseline dead reckoning model. Analysis of actual arrival traffic showed that this improved trajectory prediction accuracy leads to improved inter-arrival separation prediction accuracy for longer look ahead times. The difference in mean inter-arrival separation prediction error between the Fourier transform and dead reckoning models was 0.2 nmi for a look ahead time of 120 sec, a 33 percent improvement, with a corresponding 32 percent improvement in standard deviation.

## I. Introduction

A number of decision support tools based on aircraft trajectory modeling are in operational use in today's air traffic management (ATM) system.<sup>1,4</sup> Although the tools differ in functionality, their trajectory models face the same challenge; namely, predicting trajectories with limited aircraft intent information available from the current ATM system. Current Next Generation Air Transportation System (NextGen) Terminal Area research efforts aim to improve accuracy and reduce false alerts of legacy terminal area decision support tools by applying new trajectory prediction methods.<sup>5,6</sup> One such terminal area decision support tool is the FAA's Terminal Proximity Alert (TPA) system.

The TPA system is intended to reduce the number of compression errors (i.e., horizontal separation violations) between pairs of aircraft on the same final approach route. As a pair of aircraft, one trailing the other, approach the runway for landing, they must decelerate to their landing speed. Because the lead aircraft is closer to the runway and is, therefore, flying slower than the trailing aircraft, the horizontal separation distance between the two aircraft will begin to reduce. If the horizontal separation distance between the two aircraft reduces below a specified required threshold, then this violation is referred to as a *compression error*.

A trajectory model is utilized by the TPA system to predict aircraft separation. If separation between two aircraft is predicted to be below a specified threshold, a color-coded announcement will be displayed to the controller. The inputs to the trajectory model currently used in TPA are limited by the current ATM system data to the aircraft's current measured ground speed, distance from the runway threshold, and weight class. The aircraft's intended speed profile from its current position to the runway threshold is not known, nor is its weight or intended flap configuration. For a given weight, speed from the beginning of the approach to landing can vary by quantities on the order of 50 knots due to flap configuration changes alone.<sup>7</sup> Although some NextGen ATM concepts envision the availability of weight- and flap configuration to future decision support tools via datalink, these parameters are unavailable to current tools such as TPA.<sup>8</sup> Therein lies the challenge in predicting final approach trajectories. Today one needs methods that infer the final approach speed profile solely from the available ATM system data. Better final approach trajectory prediction methods have the potential not only to improve the performance of

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terminal area tools such as TPA, but also to improve indirectly airport surface DSTs such as the Surface Management System (SMS) which rely on accurate prediction on landing times.<sup>9</sup>

The objective of this paper is to improve the final approach trajectory model by developing new methods for inferring the intended final approach speed profile from current ATM system data. Two such methods, one based on polynomial interpolation and the other on the Fourier transform, constitute the research contribution of this paper. A description of these methods as well as wind and aircraft weight class modeling methods is presented in Section II. The methods provide an inference of the aircraft’s final approach speed profile from data of actual aircraft on final approach to Los Angeles International Airport (KLAX). The inferred speed profiles are then used to calculate final approach trajectories. The resulting trajectories are analyzed for accuracy with respect to individual predictions as well as to pairwise predictions in terms of an aircraft separation error metric (section III). The analysis includes the effect of specific modeling parameters such as wind, aircraft type, and weight class.

## II. Final Approach Trajectory Modeling Methodology

The current ATM data system lacks key final approach trajectory prediction parameters such as aircraft weight and flap configuration. This lack of aircraft intent information hinders the use of classic aeronautical engineering trajectory prediction methods. For this reason, an empirical approach to developing final approach trajectory models was taken. Subsets of actual track data recorded for aircraft on final approach to all KLAX runways during four days in early February 2010 were used as modeling data sets. The accuracy of the resulting models’ predictions was then tested on an independent analysis data set of sixteen days of KLAX final approach; this accuracy analysis is presented in Section III.

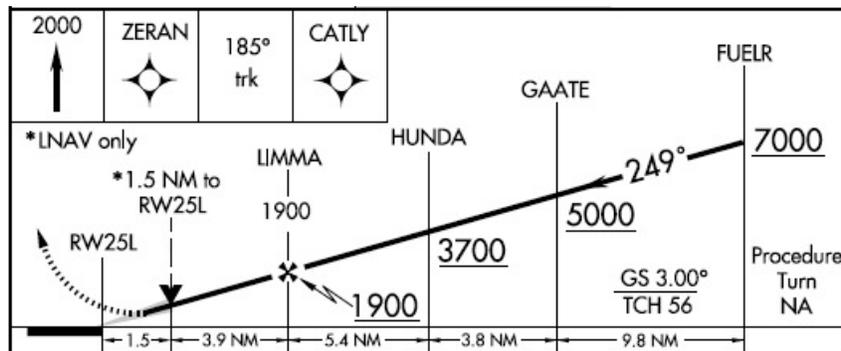


Figure 1 - Example of a Final Approach Altitude Profile

The prediction of a trajectory for the final approach phase is characterized by operational factors that both simplify and complicate trajectory prediction. A major simplifying factor is that the altitude profile is specified. Figure 1 shows an example of a published approach procedure (RNAV Rwy 25L, KLAX). By knowing the distance from the runway threshold, the aircraft’s altitude profile can be predicted by assuming the aircraft is flying the specified three-degree glide slope.

The final approach speed profile, on the other hand, is complicated by the fact aircraft must decelerate from their approach speed to their landing speed during which flap and landing gear also change configuration. Figure 2 shows the variation in ground speed data for ninety-six 737-700 aircraft from the four-day modeling data set. Aircraft decelerate from a groundspeed as high as 290 knots (kts) at the start of the approach, located 14 nautical miles (nmi) from the runway, to a landing groundspeed between approximately 120-140 kts. Although several aircraft types were studied, the analysis presented in this paper concerns mostly the Boeing 737-

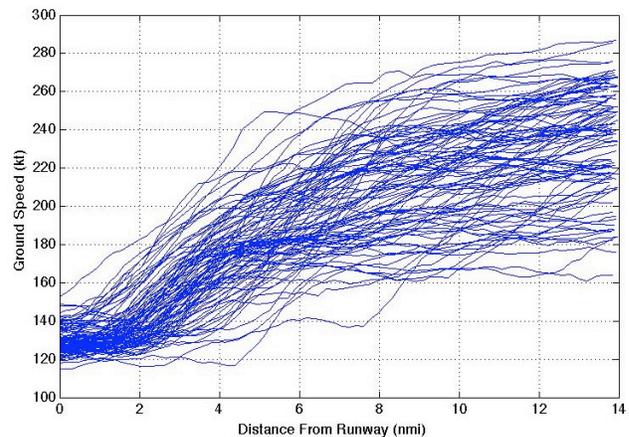
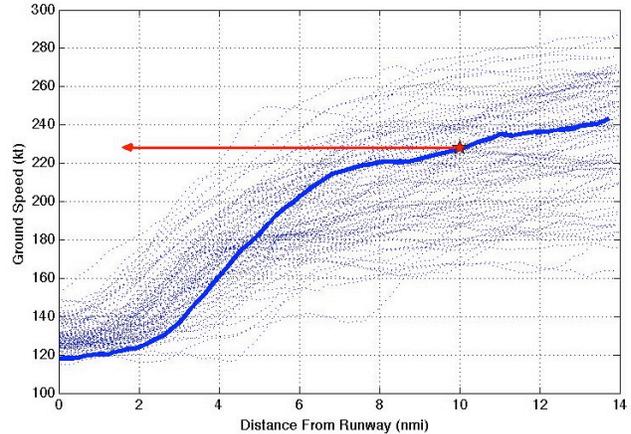


Figure 2 - Modeling data set for 737-700

700 series aircraft, one of the most common aircraft types at KLAX. For preliminary analysis of final approaches to KLAX Runway 25L, the start of final approach segment was set to correspond to the intermediate approach fix GAATE (Fig. 1), a horizontal distance of approximately 14 nmi from the runway. This final approach segment definition remained the same as the analysis expanded to include approaches to other runways. The data set was limited to “straight in” arrivals. Arrivals which turned onto the final approach route within 14 nmi of the runway are not considered.

The simplest method for inferring the final approach speed profile is dead reckoning. Typically, dead reckoning, as applied to trajectory speed profiles, assumes the aircraft will maintain its current speed for the remainder of the trajectory.

An application of dead reckoning is illustrated in Figure 3. The solid blue curve represents the speed profile of one of the ninety six 737-700 aircraft on final approach. The red star and the corresponding red line represent the dead reckoning speed profile prediction for the sample aircraft at 10 nautical miles (nmi) from the runway. One beneficial feature of the dead reckoning method is that it can adjust to the aircraft’s currently observed speed (e.g., 228 knots). By comparison, the speed at 10 nmi from the runway for any given aircraft in Figure 3 ranges from approximately 165 to 275 knots. The main shortcoming of the dead reckoning method is that the predicted speed remains constant for the rest of the trajectory, thereby failing to capture realistically a continuously changing speed profile such as that observed during final approach. Despite this shortcoming, however, the dead reckoning method is chosen here as the baseline for assessing the quality of the two methods proposed below. The rationale for this choice is that dead reckoning is widely known and well understood at the operational level.



**Figure 3 - Dead Reckoning Method**

### A. Methods for Inferring Variable Final Approach Speed Profiles

Two methods for inferring variable final approach speed profiles from current ATM system data are proposed, one based in polynomial interpolation, the other on the Fourier transform. These methods seek to combine the dead reckoning method’s ability to adjust to the aircraft’s current observed speed with the ability to model a continuously varying speed profile. The problem addressed by the methods developed here can be formulated as follows. As mentioned above, the final 14-mile portion of an aircraft’s descent path is a linear segment that follows the standard 3-degree glide slope. Parameterizing the horizontal component of that segment by the variable  $x$  which, therefore, gives the aircraft’s horizontal distance to the runway (the touchdown occurs at  $x=0$ ), the aircraft’s horizontal speed  $s$  (whether ground- or true air-) is thought of as a function of  $x$ . Consequently, the aircraft’s speed profile is of the form

$$s = s(x)$$

Among the available initial data are the aircraft’s current speed and location, henceforth denoted  $s_0$  and  $x_0$  respectively. Once the speed profile (refer to the above equation) is found, the actual descent trajectory can be found as a function  $x(t)$  of time by solving the differential equation  $x' = -s(x)$  with the initial condition  $x(0) = x_0$ . The aircraft’s initial location can be closer to the runway than 14 miles; in such cases, we have  $x_0 < 14$ .

The two methods presented here for inferring  $s(x)$  from the initial speed state  $(s_0, x_0)$  are based on a relation observed qualitatively between all pairs of curves in a representative data sample of descent speed profiles (regarded, again, as curves  $s(x)$ ). This relation is described in mathematical terms as follows. Given such a sample, one can, from knowing one curve and the initial speed state of another, completely (and with sufficient accuracy) reconstruct the latter curve. In more detail, using a number of parameter values based on the entire data sample, a transformation can be defined that infers one speed profile from another’s initial state. Therefore, it is possible to infer all the speed profiles in the sample from just one, and this inference is the objective of both the methods presented here. Estimating this one curve, henceforth called *the mid-curve*, representing “an average speed profile,” is part of tuning the parameter values for the mentioned transformation. In the first method (see Polynomial Method, section IIA-1), the mid-curve is estimated using a polynomial fit; in the second method (see Fourier Transform Method, see section IIA-2), using the Fourier transform. For both methods, one data set (*the*

modeling data set) is used to tune the parameter values, and another independent analysis data set is used to test the accuracy of the methods' predictions.

### 1. Polynomial Method

Initially, the polynomial speed profile method was developed as a groundspeed model (i.e., ignoring wind). The effects of wind in the form of a true airspeed model will be discussed later in this section. The basic methodology remains the same whether groundspeed or true airspeed is used.

Aircraft from the 737-700 modeling data set (Fig. 2) were divided into three subsets (low, medium, high) based on their speed at the beginning of the final approach, 14 nmi from the runway. A reference speed profile was then created by fitting each subset with a sixth degree polynomial. The red line in Figure 4 shows the reference speed profile created from the curve fit of the aircraft belonging to the low approach speed subset (160 - 210 kts). Sixth degree polynomials were found to provide better fit than did lower degree polynomials, without introducing the unrealistic inflections often arising in higher degree polynomials.

Once the three reference speed profiles are fit, and an individual aircraft's initial speed state ( $s_0, x_0$ ) given, these data are used to compute an applicable final approach speed profile by interpolating between the reference speed profiles. The algorithm developed and used here to carry out the above interpolation also allows for extrapolation. Future research plans, however, include computing reference curves representing the upper- and lower speed bounds for the entire 14 nmi, thus dispensing with the need for extrapolation.

Notional examples of this polynomial method are shown in Figure 5. For the example represented by the green star, the initial aircraft speed state is ( $s_0 = 230$  knots,  $x_0 = 14$  nmi). The remaining speed profile is determined by interpolating between the two lower reference speed curves represented by the solid red lines at each  $x$  from  $x_0$  to 0. The green dashed curve represent the resulting speed profile. The example shown in blue shows the resulting speed profile for an aircraft passing the 10 nmi mark. In this example, the interpolation is based on the initial condition at  $x_0 = 10$  nmi shown by the blue star. Interpolation is between the upper two reference curves for  $0 < x < 10$ . The resulting speed profile is represented by the blue dashed curve.

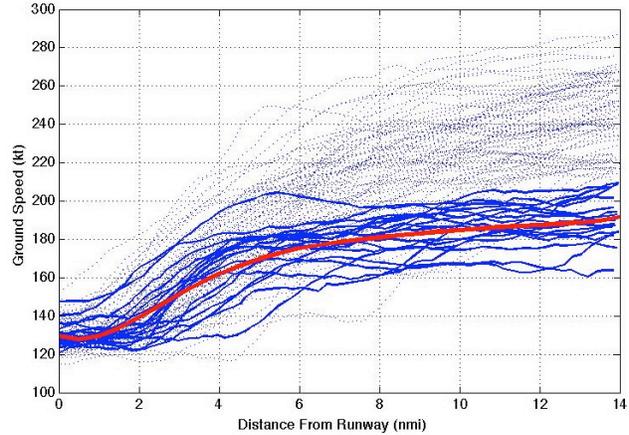


Figure 4 - Low speed polynomial fit

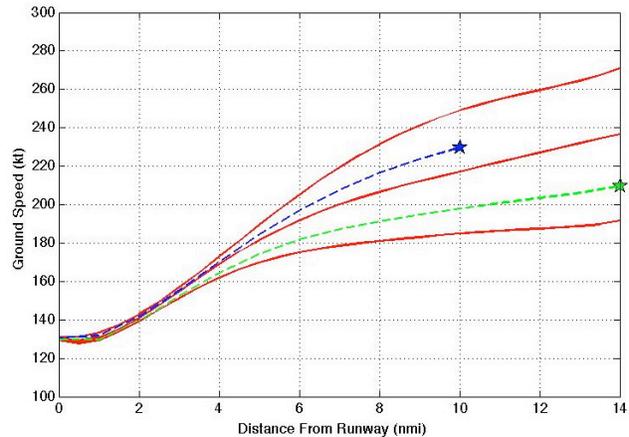


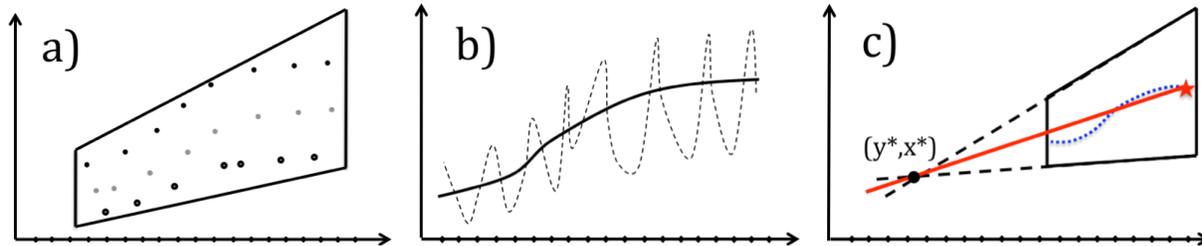
Figure 5 - Polynomial Method

### 2. Fourier Transform Method

The underlying intuitive hypothesis for this method, suggested by a visual inspection of the raw speed profile data (Figure 2), is that the difference between every two speed profiles is a function approximately linear in  $x$ . Therefore, each speed profile is sought in the form  $s(x) = s_{mid}(x) + L(x)$ , where  $s_{mid}(x)$  is the mid-curve, and  $L(x) = y^* + m(x - x^*)$  is a linear function. Here, the mid-curve  $s_{mid}(x)$  and the intercept coordinates ( $y^*, x^*$ ) are estimated using the modeling data set and are kept the same for all ensuing speed profile inferences, while the slope  $m$  is computed from the initial speed state ( $s_0, x_0$ ) of the specific aircraft whose speed profile is to be predicted.

To calculate the mid-curve, all the training data are juxtaposed (Figure 6a shows a juxtaposition of data for 3 different speed profiles) and interpolated by a single curve (see the dashed curve in Figure 6b). Using the discrete Fourier transform to filter the higher-frequency oscillations from this curve, we obtain its smoothing, which we

accept as the mid-curve (see the solid curve in Figure 6b). Since the interpolation of the juxtaposed data is generally non-periodic, which compromises the accuracy of the Fourier transform, a linear trend is calculated using MATLAB and subtracted from the interpolation, allowing a better application of the Fourier transform, and later added to the filtered data, resulting in the mid-curve.



**Figure 6 – Fourier Transform Method**

To calculate the intercept  $(y^*, x^*)$ , we form a trapezoid that “envelopes” the entire training data set (Figure 6a). The trapezoid has vertical sides at  $x = 0$  and  $x = 14$ . The endpoints of these sides are estimated so that the trapezoid envelopes the data as tightly as possible. The intersection of the lines containing the two non-vertical sides of the trapezoid is accepted as the intercept  $(y^*, x^*)$  (this is the intersection of the two dashed lines in Figure 6c).

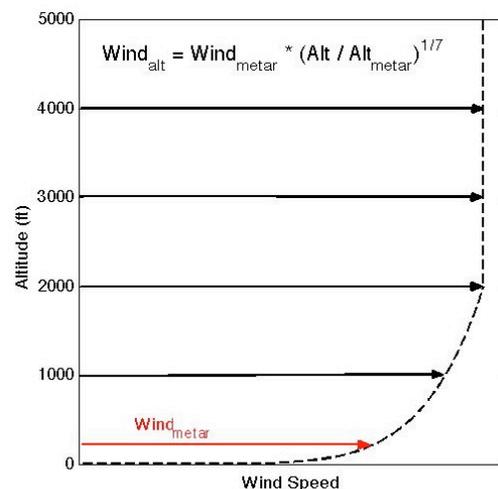
Given an initial speed state  $(s_0, x_0)$ , we take the slope to be  $m = (s_0 - y^*) / (x_0 - x^*)$ . This provides all the information needed to predict the speed profile. A sample prediction is depicted as the dotted blue curve in Figure 6c.

## B. Wind Model

The effect of winds on final approach trajectory prediction accuracy was analyzed by comparing a true airspeed based version of the Fourier transform speed profile model to its groundspeed version, which does not consider winds. The methodology for deriving a true airspeed based version of the polynomial or Fourier transform speed profile model is fundamentally the same as that described above, except that true airspeed is used in place of groundspeed. Starting with the same four day modeling data set used for the groundspeed based version (Fig. 2), true airspeed is calculated by applying a wind model based on actual METAR (a weather information format) reported surface wind speed and direction. Once this is done, the true airspeed data are expressed as a function of horizontal distance from the runway. Subsequent Fourier transform modeling steps are similar with the individual linear trend (Fig. 6c) starting at the aircraft’s initial true airspeed rather than at its groundspeed.

A benefit of using METAR reports is that they are readily available for many airports. On the other hand, the update rate for METAR winds is relatively low, typically reported on an hourly basis with occasional unscheduled reports as required. Assessment of the accuracy of METAR winds relative to other wind models, such as the Rapid Update Cycle or the Integrated Terminal Weather System, for final approach trajectory predictions is beyond the scope of this paper. However, analysis of wind effects, presented later in this paper, showed that METAR winds were adequate for this purpose.

Wind speed and direction contained in a METAR report are based on surface observations. To account for the increase in wind speed with altitude, surface winds reported by METAR were corrected using an equation that approximates the effect of the atmospheric surface boundary layer on the wind profile.<sup>10,11</sup> The resulting wind



**Figure 7 – METAR Wind Speed Profile**

profile is illustrated in Figure 7, where the surface winds increase exponentially up to the altitude of 2000 feet, above which the wind remains constant.

### **C. Aircraft Type and Weight Class Modeling**

The constant dilemma in the choice of modeling parameters is a sensible balance between two criteria, the parameter's ability to capture the behavior of the system realistically and the simplicity of the resulting model. There are over a thousand aircraft type identifiers defined by the FAA that would need to be considered in a model parameterized by aircraft type. By contrast, a trajectory model parameterized by weight class would require less aircraft-specific data, as there are currently only five weight classes specified by the FAA (Small, Large, 757, Heavy, and Super). For this reason, the effect of simplifying aircraft type and weight class modeling on final approach trajectory prediction accuracy for the 737-700 aircraft type was analyzed.

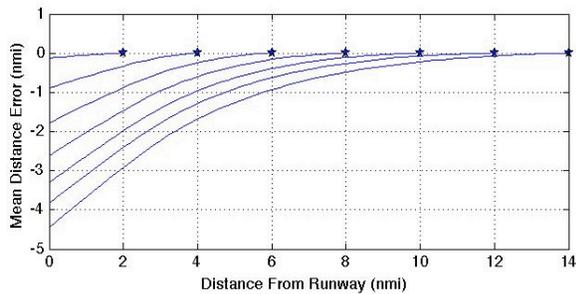
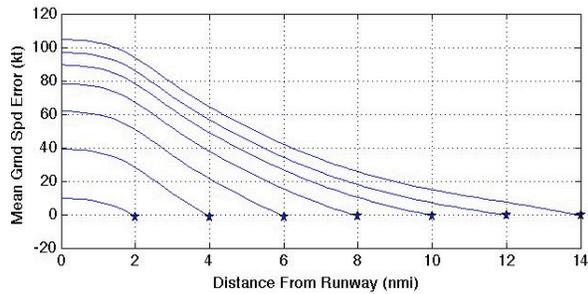
A weight class based, Fourier transform true airspeed model was created using a subset of the aircraft from the modeling data set described earlier. This subset consisted of all Large weight class aircraft (1930 aircraft), not just a single aircraft type as was the case for the 737-700 model. An "all aircraft types" (AAT) model was also created for this analysis that included 3349 aircraft of all weight classes. Once these models were created, final approach trajectories for the 737-700 aircraft from the analysis data set were calculated with each of the three Fourier transform true airspeed models (i.e., 737-700 type, Large weight class, and AAT).

## **IV. Final Approach Trajectory Analysis**

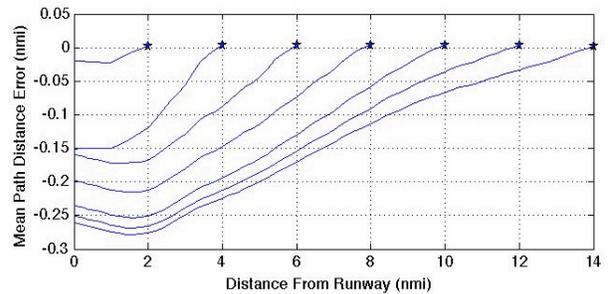
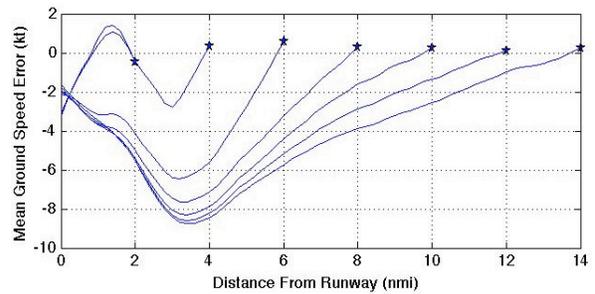
This section presents three categories of analysis for final approach trajectories derived from the various speed profile models described earlier. The first analysis category measured the accuracy of the resulting final approach trajectory for each of the speed profile modeling methods. The second analysis category evaluated the effect of key modeling parameters such as wind on final approach trajectory accuracy. The final analysis category applied operational relevant metrics such as inter-arrival separation and landing time prediction error. Aircraft from an independent set of sixteen days (mid-February to March 2010) of KLAX final approach data, different from that used to develop the models, were used for the analysis.

### **A. Final Approach Speed Profile Method Analysis**

The success of each of the three methods for inferring final approach speed profile on final approach trajectory prediction accuracy was evaluated with respect to the groundspeed based variant of each method. As with the modeling data set, the analysis data set was limited to 737-700 series straight in arrivals resulting in a total of 457 aircraft. For each aircraft, a series of final approach trajectories were calculated using initial conditions at several locations along the final approach route starting at 14 nmi from the runway and ending at 2 nmi from the runway. For each trajectory prediction made for a specified initial condition (e.g., 14 nmi from the runway), groundspeed and path distance error were calculated by holding the predicted values constant and subtracting the actual measured aircraft track values. The mean and standard deviation of the groundspeed error and path distance error for all samples were then calculated. Figure 8 shows the resulting groundspeed and path distance errors for the dead reckoning method. Each star indicates the initial condition for the corresponding trajectory prediction. For the series of dead reckoning predictions initiated for aircraft 14 nmi from the runway, mean groundspeed error increases from zero to approximately 40 knots faster than actual when the aircraft were 6 nmi from the runway, then to approximately 105 knots faster by the time the actual aircraft reach the runway. The corresponding mean path distance errors were negative, indicating the dead reckoning method on average predicts the aircraft to be closer to the runway than was actually the case.



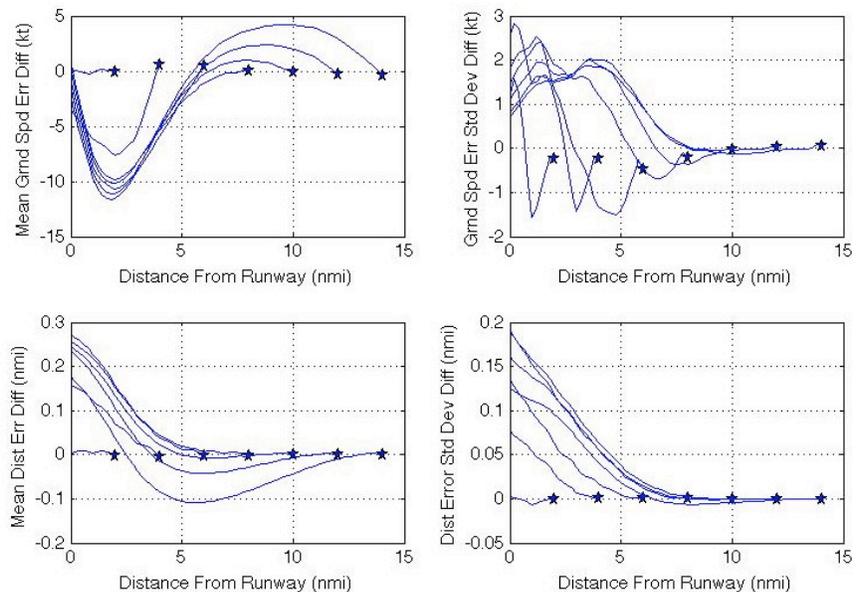
**Figure 8 - Dead Reckoning Error, 737-700**



**Figure 9 - Polynomial Method Error, 737-700**

Figure 9 shows groundspeed and path distance errors for final approach trajectories predicted with the polynomial speed profile method. The mean groundspeed and path distance error for the polynomial method was an order of magnitude less than that of the dead reckoning method. For trajectories initiated at 14 nmi from the runway, peak path distance error was approximately -0.27 nmi compared to -4.5 nmi for the same series of dead reckoning trajectories. Qualitatively, improvement in prediction accuracy was expected given the stated deficiencies of the dead reckoning method. The analysis presented here complements the qualitative expectation with a quantitative characterization of the differences.

The same procedure of error analysis was applied to trajectories predicted by the Fourier transform speed profile method. Prediction errors for the Fourier transform method were similar in magnitude to that of the polynomial method. To compare these two methods in more detail, the differences in the errors (polynomial – Fourier transform) are shown in Figure 10. The difference between the polynomial and Fourier methods were most pronounced in the latter part of the approach starting from approximately 6 nmi. from the runway. The mean path distance error for the Fourier transform method was lower than that of the polynomial at the beginning of the final approach, and higher in the latter part of the final approach. The standard deviation of the path distance error was higher for the polynomial method. Although both methods showed a significant improvement over the dead reckoning method, the magnitude of the standard deviation was on the order of the error, which indicated that both methods can be improved to account better for the variance in the actual data.



**Figure 10 - Error Difference, Polynomial Method - Fourier Method**

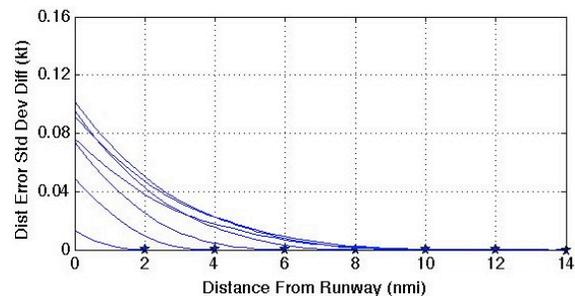
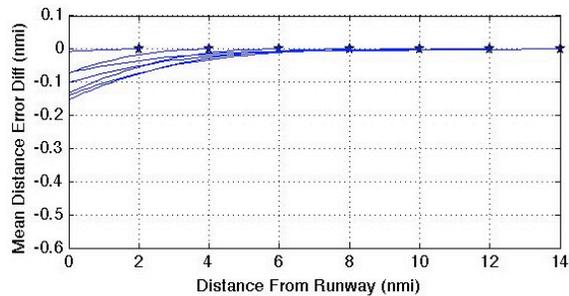
The Fourier transform method was simpler to implement than the polynomial method at this stage of development. In the modeling of other aircraft types, carried out but not presented in this paper, the polynomial method's use of multiple curves often resulted in curves that intersected or had undue inflections, the degree of which varied depending on the aircraft type and data sample size. This typically occurred in the latter part of the final approach, within 3 nmi of the runway (Fig. 5). Some manual editing of the curve fits was required in order to prevent discontinuities from occurring during the interpolation process. Because the Fourier transform method modeled the basic speed profile with one curve, it was unaffected by such anomalies. Therefore, it served as the basis for the subsequent analysis of modeling parameters and operational metrics.

## B. Final Approach Trajectory Modeling Parameters

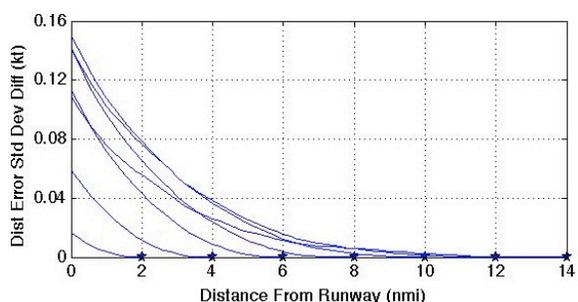
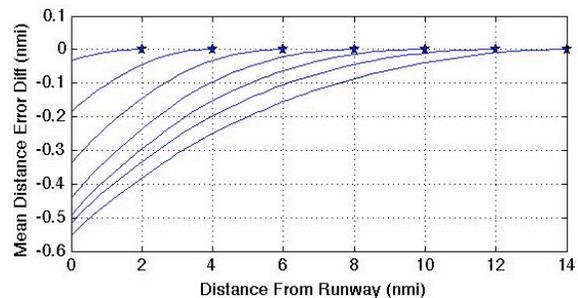
The effect of two key final approach trajectory modeling parameters, wind and aircraft type/weight class, on the accuracy of speed profile predictions are analyzed in this section. The intention of this analysis is to weigh the benefits of improved accuracy versus the additional complexity of modeling additional parameters.

### 1. Wind Effects

Final approach trajectories were calculated for all 737-700 series aircraft in the same sixteen day analysis data set described above. Trajectory errors were measured for final approach trajectories calculated with both the groundspeed and true airspeed based Fourier transform models. Figure 11 shows the difference in mean distance error and standard deviation between the groundspeed and true airspeed model for data which had a reported METAR wind between 0 and 5 knots. For 0 to 5 knots of wind, the groundspeed based model predicts the aircraft to be up to -0.15 nmi closer to the runway (i.e., more negative) than the true airspeed model predicts. It should be noted that, on average, both models predict the aircraft to be closer to the runway than actual. Standard deviation for the groundspeed model was up to 0.10 nmi more than the true airspeed model. The difference in trajectory error for data with a reported METAR wind between 10 and 15 knots are shown in Figure 12. The mean distance error difference for the groundspeed model increases from -0.15 nmi more than the true airspeed model at low wind speeds to -0.55 nmi for the higher wind speeds (10 – 15 knots). The corresponding difference in standard deviation also increases with wind speed. The standard deviation of the groundspeed model was approximately 0.15 nmi more than the true airspeed model at 10 to 15 knots of reported wind, compared to 0.10 nmi at 0 to 5 knots of wind.



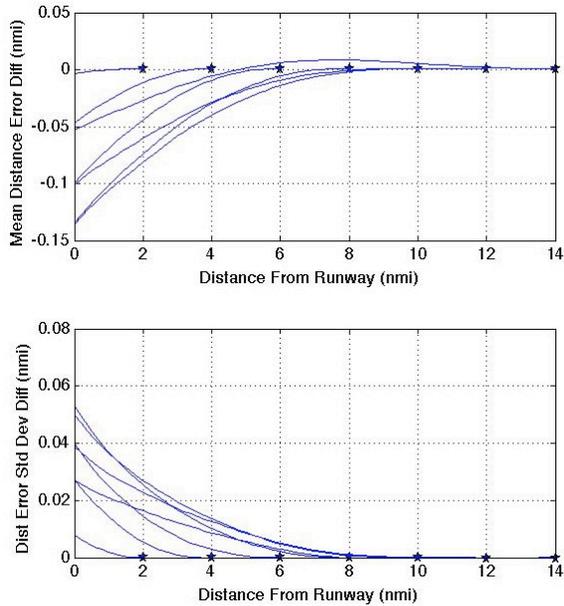
**Figure 11 - Fourier Model Error Difference, Groundspeed - True Airspeed, 0 to 5 kt winds**



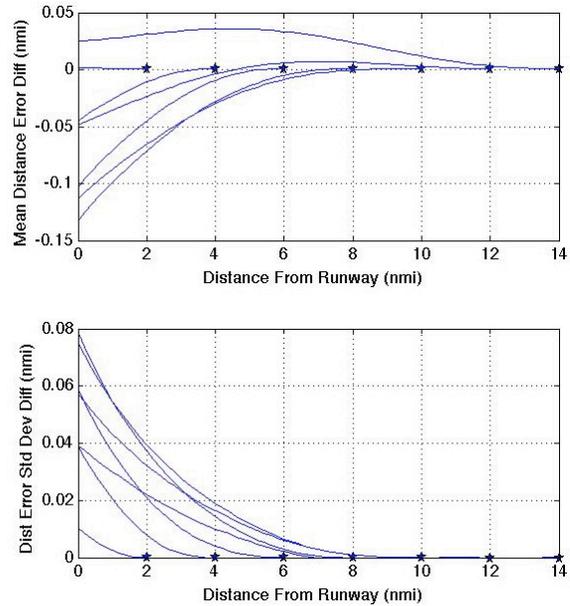
**Figure 12 - Fourier Model Error Difference, Groundspeed - True Airspeed, 10 to 15 kt winds**

### 3. Aircraft Type/Weight Class

Figure 13 shows the trajectory error and standard deviation difference between the Large weight class and the 737-700 models while Figure 14 shows the difference between AAT and the 737-700 models. The mean distance error for the 737-700 trajectories calculated with the Large weight class model was up to 0.14 nmi closer to the runway than trajectories calculated with the 737-700 specific model for the same aircraft with a max standard deviation difference of 0.05 nmi. Error difference for trajectories calculated with the AAT model was similar to that of the Large weight class model. These limited results suggest that the aircraft type specific model was more accurate than the weight class or AAT models. In addition, the number of required models offered by weight class modeling over aircraft type modeling can be further reduced by using a single AAT model without loss of trajectory prediction accuracy. Although only one aircraft type and weight class were analyzed (i.e., 737-700), these results are not expected to change when the analysis of other aircraft types and weight classes are performed.



**Figure 13 - Fourier Model Error Difference, 737 type - Large Weight Class**



**Figure 14 - Fourier Model Error Difference, 737 type - All Aircraft Types**

## C. Operational Metrics

The motivation for developing improved final approach trajectory models is, ultimately, to improve the operational performance of trajectory based decision support tool. In a broad sense, these tools assist controllers in performing two key tasks; maintaining separation between aircraft (e.g., TPA) and scheduling (e.g., SMS). The operational effects of final approach trajectory modeling on these two tasks were measured with the following two metrics; inter-arrival separation error and landing time error.

### 1. Inter-arrival Separation Error

The first step in analyzing inter-arrival separation error was to identify and calculate trajectories for all arrival pairs in the sixteen day analysis data set. An arrival pair was defined as two aircraft that were both within 14 nmi of and landing consecutively at the same runway, resulting in 1505 arrival pairs. For each arrival pair, a series of final approach trajectories were calculated with an all types based Fourier transform model. An AAT model was used in order to minimize the number of models required to account for the large number of aircraft type pairings inherent to this type of analysis. Dead reckoning trajectories for the same pairs and initial conditions were calculated and used as a baseline for comparison.

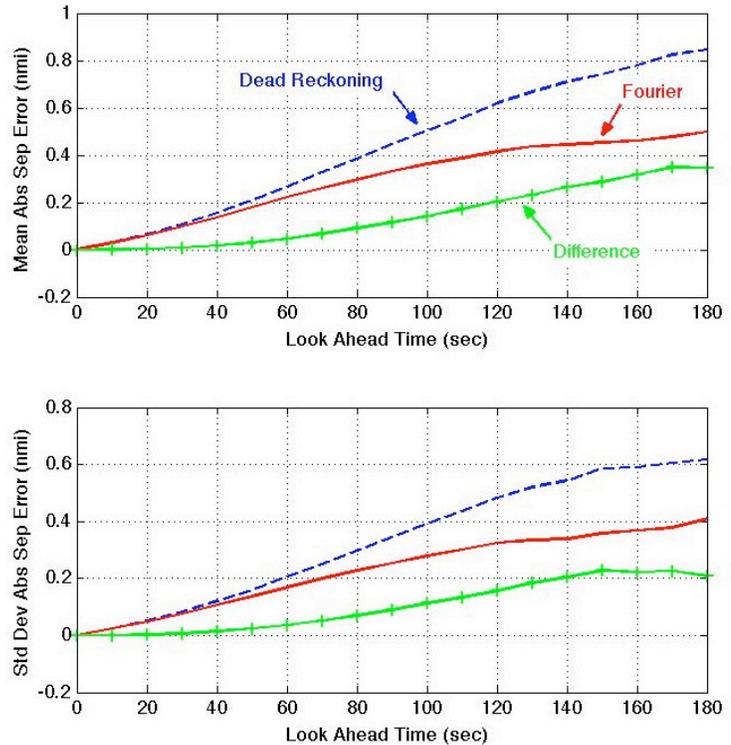
Inter-arrival separation error was measured as a function of look ahead time and the trailing aircraft's distance from the runway. Once a given pair of trajectory predictions for a specified initial condition (e.g., trailing aircraft at 14 nmi from the runway), separation error was determine by holding the predicted separation to the lead aircraft

constant and subtracting the actual measured aircraft pair separation at each subsequent track time step. Look ahead time is defined as the forecast time of the prediction.

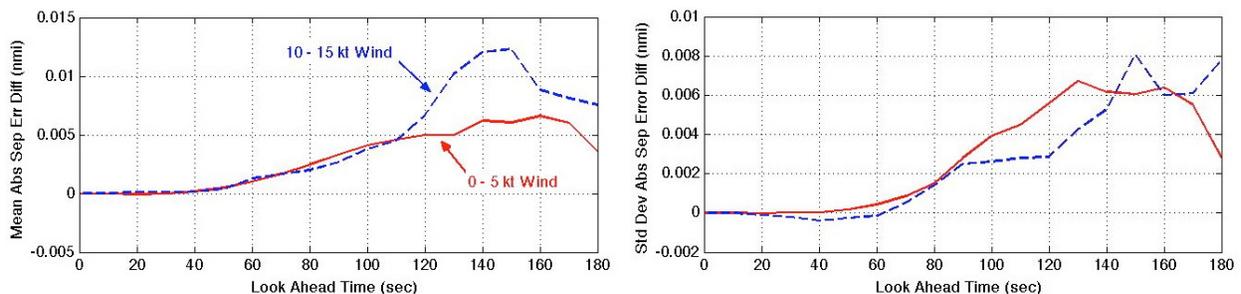
Figure 15 shows the mean and standard deviation of separation errors for the Fourier transform and the dead reckoning models for final approach trajectory predictions initiated when the trailing aircraft was 14 nmi from the runway. The mean and standard deviation of the separation error increased with look ahead time for both the Fourier and dead reckoning models. The difference in mean inter-arrival separation prediction error between the Fourier transform and dead reckoning models increased with increasing look ahead times from 0.03 nmi (14 percent), for a look ahead time of 45 sec to 0.2 nmi (33 percent) for a look ahead time of 120 sec. The corresponding reductions in standard deviation were 15 and 32 percent, respectively.

To put these results in context, minimum required inter-arrival separation for certain weight class combinations is no less than 3.0 nmi. The FAA's TPA tool currently alerts controllers of pending losses of separation for look ahead times of 22 and 45 seconds. The operational benefit of increased accuracy over longer look ahead times, however, may not become apparent until strategies for mitigating false alerts are considered. For example, the CTAS enroute conflict detection tool applies an algorithm which only alerts the controller of a pending conflict after a specified number of consecutive positive conflict detections have occurred.<sup>12</sup> This functionality is analogous to a controller's monitoring two aircraft for a period of time before taking corrective action.

The method used here for evaluating the effect of wind on inter-arrival separation error was similar to that described above. In this case, a Fourier groundspeed which did not model winds was compared to a true airspeed model that accounted for winds. The results shown here include only trajectory predictions initiated when the trailing aircraft was 14 nmi from the runway. Figure 16 shows a difference of less than 0.005 nmi in the mean and standard deviation of the separation error due to wind conditions for look ahead times less than 100 sec. These results were, arguably, expected because both aircraft in the arrival pair would be flying in the same wind field. Hence, wind effects on the pair of aircraft effectively cancel each other out.



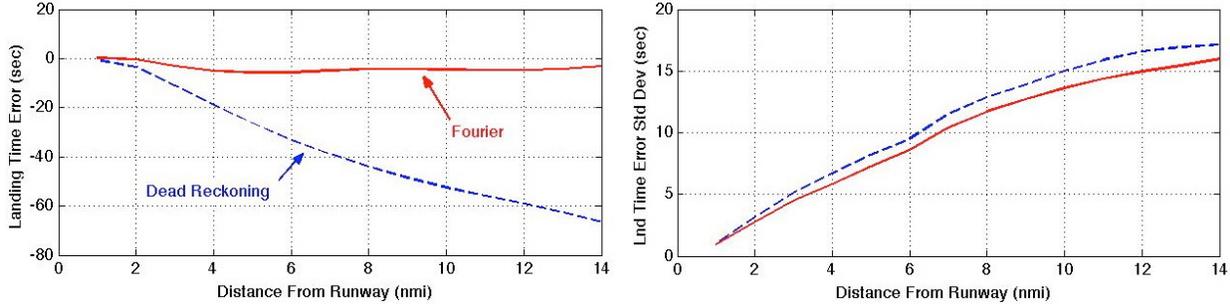
**Figure 15 – Inter-Arrival Separation Error, Dead Reckoning Method versus Fourier Method**



**Figure 16 - Inter-Arrival Separation Error difference due to wind, Fourier Groundspeed -True Airspeed Models**

## 2. Landing Time Error

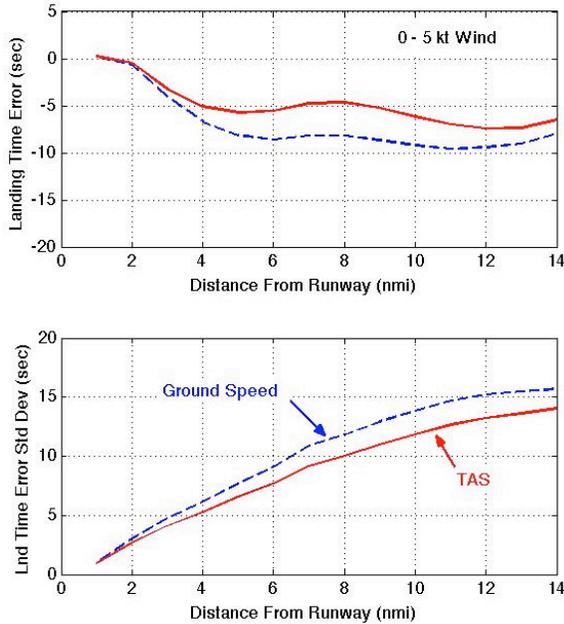
The landing time error metric was intended to measure the effect of final approach trajectory modeling on scheduling type tasks. The same sixteen day analysis data set as above was used in this analysis. However, only 737-700 series aircraft were considered. For each aircraft, final approach trajectories were calculated for a series of specified distances from the runway. Landing time error for each trajectory was then determined by subtracting the predicted landing time from the actual landing time.



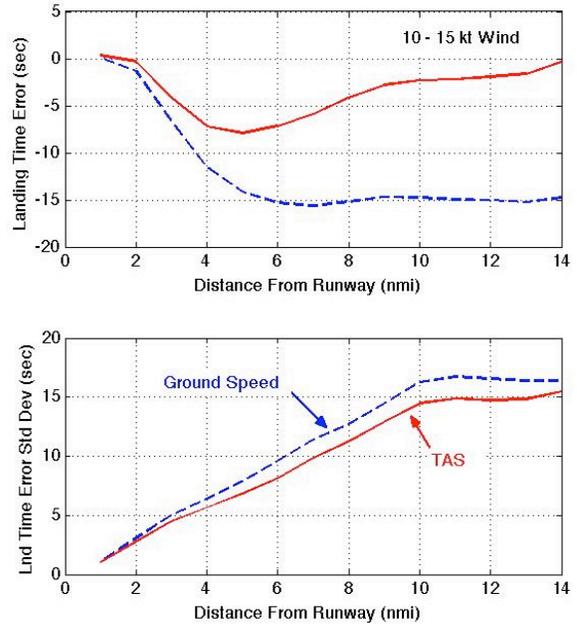
**Figure 17 – Landing Time Error, Dead Reckoning Method versus Fourier Method**

Landing time error for the dead reckoning and Fourier transform method are shown in Figure 17. Unlike with the inter-arrival separation error results for these two methods (Fig. 15), the landing time prediction accuracy was significantly better for the Fourier method. Mean landing time error was less than approximately 5 seconds earlier than actual for all Fourier trajectories. By comparison, the dead reckoning method had a mean landing time of approximately 66 seconds earlier for prediction initiated at the start of the final approach, 14 nmi from the runway, and a mean error of approximately 20 seconds earlier for predictions initiated at 4 nmi from the runway. The difference in standard deviation for the two methods were less than 1.5 sec, but relatively large, as high as 17 sec, compared to the mean. This was an indication of the wide variation in the actual landing time among aircraft of the same type.

The effect of wind on landing time error was also analyzed. Landing time errors were found to decrease with the use of a true airspeed based model instead of a groundspeed based model. For wind conditions between 0 and 5 knots (Figure 18), the difference in mean landing time error between the Fourier groundspeed and true airspeed



**Figure 18 – Wind Effect on Landing Time Error, Fourier Groundspeed versus True Airspeed Model, 0 to 5 knots Wind**



**Figure 19 – Wind Effect on Landing Time Error, Fourier Groundspeed versus True Airspeed Model, 10 to 15 knots Wind**

models was less than 5 seconds for all predictions. However, the difference in mean landing time error between the groundspeed and true airspeed model increases to nearly 15 seconds in wind conditions between 10 and 15 knots (Figure 19). The difference between the standard deviation of the landing time errors of in the two models remain similar for all wind conditions.

## V. Conclusions

The limited aircraft intent information available from the current ATM system for predicting final approach trajectories poses a challenge for efforts to improve trajectory based decision support tools. The research presented in this paper proposes two new methods for inferring the speed profile from limited ATM system data necessary for final approach trajectory prediction. Analysis of these methods, one based on polynomial interpolation and the other based on the Fourier transform, showed better final approach trajectory prediction accuracy than that of the dead reckoning method.

This analysis included a study of the key final approach trajectory modeling parameters of wind and aircraft type/weight class were also studied. The results showed the application of a METAR based wind model improved trajectory prediction accuracy and decreased standard deviation as the reported wind speed increased. Mean path distance error difference between the groundspeed and true airspeed model for winds between 0 and 5 knots was -0.15 nmi compared to -0.55 nmi for winds between 10 and 15 knots.

A study based on a single aircraft type (i.e., Boeing 737-700) showed that an aircraft type specific model offered improved trajectory prediction accuracy over a weight class based model. The benefit of a weight class based model over one based on aircraft type is the considerably lower number of different category values: there are only five distinct weight classes, but there are thousands of aircraft types. Furthermore, the analysis showed the number of models necessary can further be reduced to one model accounting for all aircraft types without loss of accuracy.

Additional analysis was conducted to assess the operational benefits of final approach modeling methods. An analysis of actual arrival traffic showed inter-arrival separation prediction accuracy of the Fourier transform model was increasingly better than that of the predictions made by the dead reckoning model as look ahead time increased. The difference in mean inter-arrival separation prediction error between the Fourier transform and dead reckoning models was 0.03 nmi for a look ahead time of 45 sec (time of initial TPA warning), but increased to 0.2 nmi for a look ahead time of 120 sec. The operational benefit of increased accuracy over longer look ahead times, however, may not become apparent until strategies for mitigating false alerts are considered. The modeling of winds did not improve inter-arrival prediction accuracy for the same look ahead times.

Landing time prediction accuracy did benefit from the improved trajectory prediction accuracy associated with the Fourier transform model and wind modeling. The Fourier transform model had mean landing time errors less than 5 seconds while the dead reckoning method had landing time errors greater than 60 seconds. A true airspeed based model improved landing time prediction accuracy over a groundspeed based model as the reported wind speeds increased. Landing time error difference between the groundspeed and true airspeed model increased from approximately 5 seconds for winds between 0 and 5 knots to 15 seconds for winds between 10 and 15 knots.

## VI. References

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- <sup>2</sup>Common ARTS Computer Program Functional Specification (CPFS) Conflict Alert, NAS-MD-632, U.S. Department of Transportation, Federal Aviation Administration, April 2007.
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