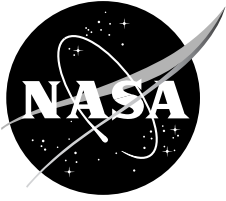


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A Path-Stretch Algorithm for Conflict Resolution

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March 2009

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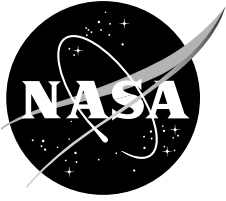
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A PATH-STRETCH ALGORITHM FOR CONFLICT RESOLUTION

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I. INTRODUCTION

Consider two aircraft that are first predicted to be in conflict at some instant between 2 and 20 minutes after the current time. (A conflict occurs when horizontal separation is less than 5 nmi when vertical separation is less than 1000 ft.) An effective resolution of the conflict can be for one aircraft to implement a “path stretch,” a maneuver that involves a turn from its present course with a turn back to a downstream waypoint, while achieving a specified delay. Delay is defined as the difference in the estimated times to the return waypoint along the resolution path and the original path. The maneuver may be started while the aircraft is in climb, cruise, or descent, and should be independent of the vertical profile defined by its flight plan.

A path-stretch maneuver is particularly useful when two aircraft are “in trail” when the leading aircraft is slower, or when descending arrivals on converging routes are predicted to be in conflict near the arrival fix. While this maneuver can provide a specified delay if flown properly, there is no guarantee that a specified separation will be achieved between the conflicting aircraft. Even if the maneuver clears the conflict, the separation achieved may be excessive, or the conflict may recur at a downstream location. Therefore, it is essential that the maneuver-generation process be part of a decision-support system that checks each “trial-plan” maneuver for conflicts before a clearance is issued to the aircraft.

This monograph documents a simple and reliable path-stretch algorithm included with a suite of automated resolutions for the Airspace Concept Evaluation System (ACES), an air-traffic simulation (ref. 1). The suite is part of the Advanced Airspace Concept (AAC), described by Erzberger (ref. 2). Another algorithm, useful for resolving level conflicts, is described in a companion monograph (ref. 3). Work is also in progress to add the AAC algorithms to the Center/TRACON Automation System (CTAS) (ref. 4). This paper presents the path-stretch algorithm along with a procedure for turning back to a waypoint to resume the original flight plan.

The paper begins with a review of the path-stretch scenario, followed by a description of the basic algorithm. The logic for choosing a turn, a turn-back point, and a return waypoint to achieve a given delay is covered. Next, the inclusion of constant-radius turns is described. Subsequent sections discuss how the algorithm fits into the AAC-ACES interface along with a case study to illustrate its use, and finally, concluding remarks. An appendix reviews the rhumbline equations used in the algorithm to compute heading and distance between points on the Earth defined by latitude and longitude parameters.

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II. BASIC “PATH-STRETCH” IDEA

A useful starting point for describing a path-stretch maneuver is to consider the flight-path horizontal projection shown in figure 2.1. The aircraft is flying in an easterly direction, where the small circle indicates the initial position, the dashed line indicates the intended flight path, and each small square identifies a waypoint. The point of separation loss is marked with an asterisk (the predicted path for the other aircraft in the conflict is not shown). A path-stretch maneuver will consist of legs L_V (a turn of an angle υ from the current heading) and L_R (a turn back to a candidate downstream waypoint). The legs form a triangle with base R (a line from the initial point to the return waypoint). Note that all “straight” lines in figure 2.1 are actually rhumb lines on the Earth surface (see appendix A).

Preferred operational procedure requires that $L_V \leq L_R$. Thus, the maximum path-stretch length available for a given turn and return waypoint would be obtained when an isosceles triangle is formed with the base R ($L_V = L_R$). Although the airspeed along each leg is assumed constant, the wind field encountered will affect the groundspeed, so the delay achieved in this case may not be maximum. However, the $L_V = L_R$ case is a useful way to proceed: If the required delay by a path stretch to a given return waypoint is *less* than that attained by the equal leg-length path, the required delay can be found by “sliding” the turn-back waypoint down the vector leg toward the initial point.

Selection of Candidate Waypoints

To complete a path stretch, the maneuvering aircraft must return to its original flight path, preferably to a designated flight-plan waypoint. To be a candidate for return, a waypoint must have a line-of-sight (LoS) range from the initial point within suitable limits. The minimum range is set to twice the distance to separation loss, a distance usually available in the conflict data record. The maximum range is set to the lesser of the range to the final fix and 500 nmi. The final fix is excluded unless there is no other candidate. If no flight-plan waypoint exists within the range window, one or more waypoints may be inserted and the one ultimately selected added to the flight plan.

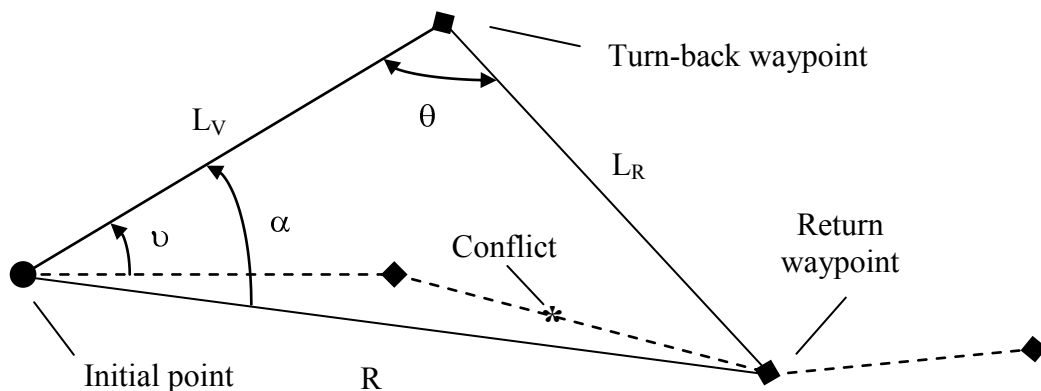


Figure 2.1. A path-stretch maneuver with legs L_V and L_R and base R .

Calculation of Maximum Delay

With return waypoints selected as indicated previously, turns will be chosen from a set, say, 15, 30, or 45 degrees (in either direction) for each aircraft, and path-stretch trial plans for conflict resolution will be created to achieve delays of, for example, 1.5, 3.0, and 5.0 minutes. However, not every turn-waypoint combination will be capable of yielding a maneuver with the desired delay. It is helpful to calculate and store the maximum delay ΔT for each rhumbline isosceles triangle associated with a given turn-waypoint pair prior to creation of trial-plan maneuvers.

The procedure for finding a triangle with two equal sides of length L starts by using the heading X and length R of the base leg to calculate the angle α (see figure 2.1):

$$\alpha = X_V - X \quad (2.1)$$

where X_V , the heading of the vector leg, is given by

$$X_V = X_I + \upsilon \quad (2.2)$$

where X_I is the initial heading. A Newton-Raphson algorithm has been devised to yield the position of the turn-back point using a starting value for the heading of the return leg

$$X_R = X - \alpha \quad (2.3)$$

that is based on a planar isosceles triangle. With the resulting length L , the maximum delay obtainable for a given turn-waypoint pair is estimated from

$$\Delta T = L * (1/V_V + 1/V_R) - T_C \quad (2.4)$$

where V_V and V_R are the groundspeeds predicted along the paths, and T_C is the predicted time at the candidate waypoint for the original trajectory.

III. CALCULATING THE DESIRED DELAY

For a vector turn υ and return waypoint that can yield a desired delay $\tau < \Delta T$, an estimate of the length of the vector leg L_V that will realize that delay can be made. If the time to reach the return waypoint along the original path is T_C , the time along the stretched path should be

$$T_R = T_C + \tau \quad (3.1)$$

and, referring to figure 2.1, use the cosine law with the planar approximation to obtain

$$L_V = [R^2 - (VT_R)^2] / [2(R \cos \alpha - VT_R)] \quad (3.2)$$

where V is the average groundspeed along the path. This value will serve as a starting value for calculating the vector leg of the path-stretch maneuver, unless it is greater than some maximum (say 100 nmi). In that case, choose a larger vector turn (υ is selected from a fixed set of values, e.g., 15, 30 or 45 degrees), and calculate a new L_V .

Now, with the vector-leg heading X_V and starting value of L_V , perform the following steps:

1. Determine the position of the turn-back waypoint $(\varphi, \lambda)_{tb}$ using eq. (A.5) or eq. (A.6) with $L = L_V$ and $(\varphi_1, \lambda_1) = (\varphi, \lambda)_I$ (the initial position).
2. Calculate the return vector (X_R, L_R) from the turn-back waypoint to the return waypoint $(\varphi, \lambda)_{rt}$, using eqs. (A.1) – (A.3). If $|X_R| > 135^\circ$, try the next waypoint.
3. Estimate the delay accumulated along the stretched path from the initial point to the return waypoint as

$$d = L_V / V_V + L_R / V_R - T_R \quad (3.3)$$

where T_R is the estimated time at the return waypoint, and V_V and V_R are the groundspeeds along the legs, using the estimated winds and the average of the airspeeds at the initial and return points.

4. Calculate the delay error $\varepsilon = d - \tau$ and use a Newton-Raphson procedure with iteration of steps 1 – 3, varying L_V until the error is sufficiently small.

If the desired delay $\tau > \Delta T$ for a given turn-waypoint combination, then a waypoint farther downstream (if there is one) can be tried with the same vector turn, or, a larger turn (if possible) can be tried with the same return waypoint. Note that each trial maneuver should be checked for conflicts by a decision-support system. Once a maneuver is predicted to be conflict free, a clearance to perform the maneuver can be issued to the aircraft.

Accounting for Winds

Clearly, the delay predicted for a path-stretch maneuver will be closer to that realized in practice if a reasonable estimate of the wind field can be used in the algorithm described earlier. This estimate may not be so important for conflicts predicted during en route portions of flight, since each trial maneuver will have been checked for conflicts. However, for aircraft in an arrival stream that are being sequenced, accurate delay estimates for conflict resolutions are important, so the path-stretch algorithm has been written to account for the wind field along the aircraft trajectories.

Application of the algorithm requires that the conflict-detection data include the aircraft “state” at *each* flight-plan waypoint, or enough information to predict it. This state includes time, latitude, longitude, altitude, airspeed, heading, and winds. For each turn-waypoint pair, airspeed, wind speed and direction are averaged and assumed constant over the stretched path, except during the initial and final turns. Because the ground track along the path is known, the ground speed along each leg can be computed from

$$V_G = V_A \cos \beta + V_W \cos \eta; \quad \beta = \sin^{-1}(V_W \sin \eta / V_A) \quad (3.4)$$

where V_A is airspeed, V_W is wind speed, and η is the angle between the wind direction and ground track. It is assumed that $V_W < V_A$.

IV. ADDING “INSIDE” TURNS

It is relatively straightforward to modify the procedure described in the previous sections to include constant-radius turns with the maneuver specification. This inclusion should serve to improve the correspondence between the requested trial plan and the maneuver actually performed. The situation is illustrated in figure 4.1, which shows that when heading for a downstream waypoint, the turn to the next heading is initiated before that waypoint. Hence the turn at point TB to the return-leg heading and the turn at point RT to the flight-plan heading are called “inside” turns.

If ν_1 is the turn from the initial heading to the vector heading, let the subsequent turns to the return heading and the flight-plan heading be ν_2 and ν_3 , respectively. The arc lengths for the three (coordinated) turns are then

$$L_1 = r\nu_1, L_2 = r\nu_2, L_3 = r\nu_3, \text{ where } r = V_A^2 / (g \tan \phi) \quad (4.1)$$

In eq. (4.1), V_A is the average airspeed, ϕ is the (constant) bank angle, and g is the gravitational constant. The straight portions of the vector and return legs are given by

$$L_{VS} = L_V - (l_1 + l_2); L_{RS} = L_R - (l_2 + l_3) \quad (4.2)$$

where

$$l_1 = r \tan(\nu_1 / 2), l_2 = r \tan(\nu_2 / 2), l_3 = r \tan(\nu_3 / 2) \quad (4.3)$$

and where L_V is the distance between points p and tb and L_R is the distance between points tb and rt. Note that point p is a distance l_1 from the initial point, while the maneuver rejoins the flightplan route a distance l_3 from point rt.

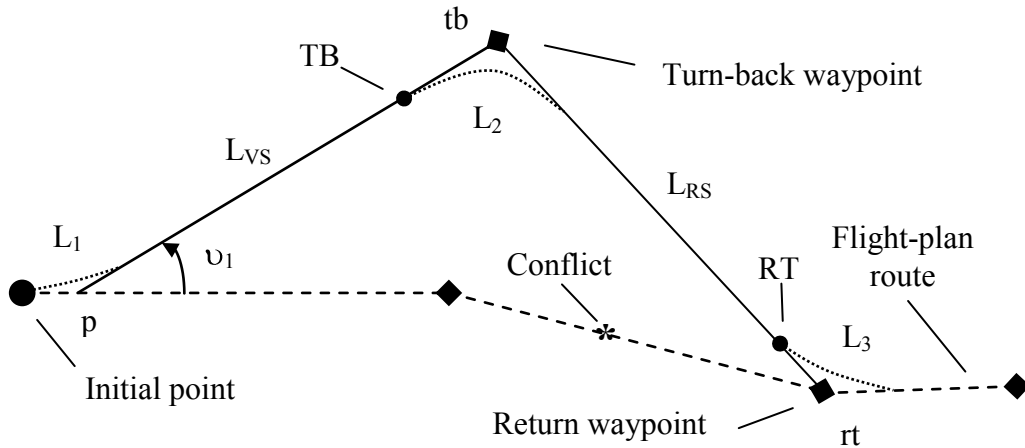


Figure 4.1. A path-stretch with straight segments L_{VS} and L_{RS} and turns L_1 , L_2 , and L_3 .

Now the delay for the stretched path is computed as

$$d = L_1 / V_1 + L_{VS} / V_V + L_2 / V_2 + L_{RS} / V_R + L_3 / V_3 - (T_R + l_3 / V_F) \quad (4.4)$$

where V_1, V_2, V_3 are the average ground speeds along the turns and V_V, V_R are the ground speeds along the straight segments L_{VS} and L_{RS} . Note that the time to rejoin the flight path has been added to the return time (V_F is the groundspeed on the flight path near point rt). The same iterative procedure for achieving a desired delay outlined in the previous section can be used, except that eq. (4.4) is used to calculate the delay.

V. A CASE STUDY

The path-stretch algorithm described in the previous sections has been developed and tested in an environment with MATLAB™ software from the MathWorks, Inc.; it was then converted to Java, and implemented with the AAC auto-resolution software in the ACES air-traffic simulation. For the tests being conducted at Ames Research Center, the simulation uses flight-plan data from one day of flights in the Cleveland airspace. The aircraft start from their respective airports at scheduled departure times, and fly according to their filed flight plans. Each aircraft within the Cleveland Center is checked for conflict with all other aircraft in the center every two minutes, and a conflict list is sent to the AAC auto-resolution module. Figure 5.1 shows the ACES–AAC interface.

This section provides a case study of one conflict pair for which several trial plans are created. The trial plans consist of path-stretch maneuvers for one aircraft, which would be created in the AAC module, and ordered so that the maneuver with the least delay from the original route would first be sent to ACES to be checked for feasibility and conflicts. This study, however, was performed with MATLAB using the algorithm described in the previous sections applied to ACES conflict data. The same algorithm, with some practical constraints, has been implemented in Java for the AAC module.

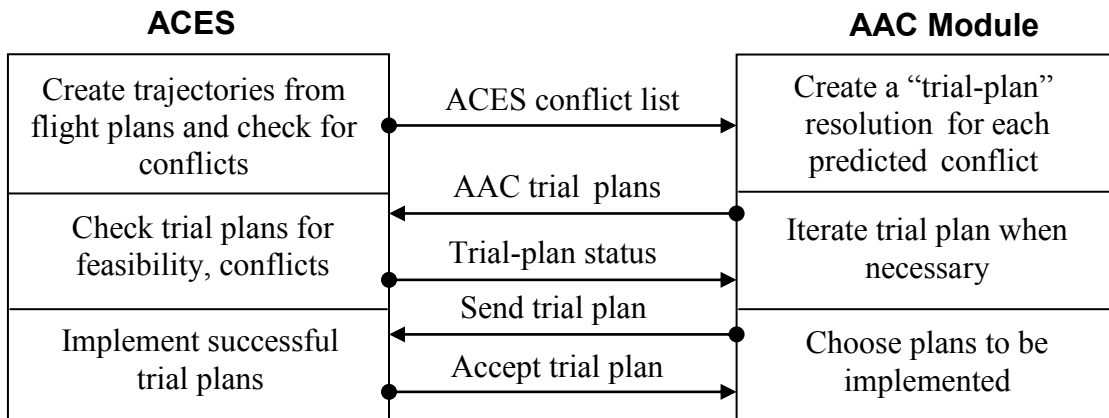


Figure 5.1. Interface of ACES and the AAC auto-resolution module.

The case chosen for this paper is a conflict between flights AAL309 (an MD-80) and UAL8193 (a B757), flying at 31,000 ft through the Cleveland Center, both en route to Chicago. The data record accompanying each conflict detected by ACES and sent to the AAC module includes, for each aircraft, position, velocity, and time at the initial point; the first-loss-of-separation point; the minimum-separation point; and each flight-plan waypoint. Data for the initial point should be considered “measured,” the rest, “predicted” by the ACES trajectory generator. A summary of the conflict data is given in table 5.1.

Calculations of LoS between the aircraft at the initial time yields $S_0 = 9.1$ nmi. Further examination of the data reveals that the aircraft have all flight-plan waypoints in common: i.e., they are “in trail”, only 9.1 nmi apart initially. The speeds differ by about 45 kn: The UAL flight is predicted to overtake the AAL flight in 6.3 min, passing with a minimum separation (MS) of 0.2 nmi if no action is taken. Turning and delaying the slower AAL aircraft to let the UAL aircraft be ahead when it passes the AAL return waypoint would probably be the first choice of an air-traffic controller. Here the path-stretch algorithm will be used to construct maneuvers for the AAL aircraft with predicted delays of 1.5, 3.0, and 5.0, minutes using both right and left turns of 15, 30, and 45 degrees.

First the candidate return waypoints are determined and each waypoint-turn combination tagged with its maximum delay. The distance to loss of separation for AAL309 is 40.3 nmi; its range to the final fix is 152 nmi: The range from the initial point to a candidate waypoint must be $80.6 < R < 152.0$ (Section II). There are nine flight-plan waypoints. The distances found using (A.1) – (A.4) show that waypoints 3 – 8 satisfy the range criteria. Next the maximum delay for each valid waypoint with each turn (in either direction) is calculated and stored. The results are shown in table 5.2, where it is seen that neither 15° turn can produce even the smallest desired delay: the maximum delay available is only 1.0 minute (left turn, returning to waypoint 7 or 8).

Finally, using the results shown in table 5.2, the path-stretch maneuvers can be created. For each delay value τ (starting with the smallest), a turn is selected (starting with the smallest), and the waypoints are checked (in ascending order) to see if $\tau < \Delta T$. The first waypoint for the given vector that satisfies this criterion will serve as the return point for calculation of path legs L_V and L_R , using eq. (4.4) and the iterative procedure outlined in Section III. If no waypoint satisfies the criterion, the next turn is selected (if possible), and the waypoints are checked again. The algorithm results for this case study are shown in table 5.3.

TABLE 5.1. CONFLICT DATA FOR PATH-STRETCH CASE STUDY

	Initial $T_0 = 0$			First Loss $T_{FL} = 5.5$ min			Min Separation $T_{MS} = 6.3$ min		
	Alt, ft	Spd, kn	Hdg, deg	Alt, ft	Spd, kn	Hdg, deg	Alt, ft	Spd, kn	Hdg, deg
AAL	31000	438.8	-107.0	31000	439.0	-106.7	31000	439.0	-105.7
UAL	31000	483.5	-106.9	31000	485.0	-106.9	31000	485.0	-106.9

TABLE 5.2. MAXIMUM DELAY ΔT (MINUTES) AT VALID RETURN WAYPOINTS (AAL309)

wpt	-15°	+15°	-30°	+30°	-45°	+45°	Spd, kn
3	0.5	0.3	2.0	1.7	5.1	4.5	438.7
4	0.6	0.4	2.3	1.9	5.9	4.9	438.6
5	0.7	0.4	2.7	2.0	6.9	5.3	438.6
6	0.9	0.4	3.2	2.1	8.2	5.7	438.6
7	1.0	0.4	3.5	2.2	14.4	8.9	438.6
8	1.0	0.4	3.8	2.3	15.3	9.6	438.6

TABLE 5.3. SUMMARY OF PATH-STRETCH MANEUVERS WITH DESIRED DELAYS

τ , min	ν_1 , deg	L_V , nmi	ν_2 , deg	L_R , nmi	ν_3 , deg	wpt	MS, nmi*
1.5	-30	45.6	+54.2	58.0	-18.4	3	8.2
1.5	+30	48.7	-55.4	54.9	+31.2	3	8.1
3.0	-30	73.5	+63.1	78.1	-29.1	6	8.2
3.0	+45	49.4	-76.5	65.5	+37.3	3	7.9
5.0	-45	65.2	+90.9	65.6	-40.1	3	8.0
5.0	+45	75.1	-84.9	77.7	+48.9	5	7.9

* MATLAB trajectory comparison

It should be noted that the MATLAB resolution software includes a level conflict check. Since this case represents a *level* conflict scenario, the trajectory paths of the two aircraft could be tested for separation every 5 sec for each trial plan, with a look-ahead time of 12 min. Although no other aircraft are included, this check is useful for monitoring resolution performance. In the column labeled “MS” it is seen that all turns appear to meet the requirement that the minimum separation be greater than 5 nmi (conflict free), at least for the first 12 minutes.

The turn of 45° for the AAL309, shown in the plan view of figure 5.2, results in a delay of 5.0 min and allows the faster UAL aircraft to pass in front. In the ACES-AAC implementation, however, the delay of 1.5 min with the -30° vector turn would be tried first. If that maneuver were conflict free with respect to **all other** aircraft in its vicinity, the resolution would be accepted, and the next conflict considered. If it were not, then the delay of 1.5 min with the 30° vector would be tried next. The trial maneuvers would be sent to ACES in the order indicated in table 5.3.

VI. CONCLUDING REMARKS

This paper has documented a simple and reliable path-stretch algorithm that has been included with a suite of automated resolutions in an advanced air-traffic simulation. The long-term goal of this work is to extend the Advanced Airspace Concept for use in the real-time Center-TRACON Automation System (ref. 4). The present paper outlines the theory and application of the algorithm, which includes turning back to a waypoint to resume the original flight plan. The path-stretch trial plan with the smallest delay and turn vector that provides a conflict-free path is the resolution chosen.

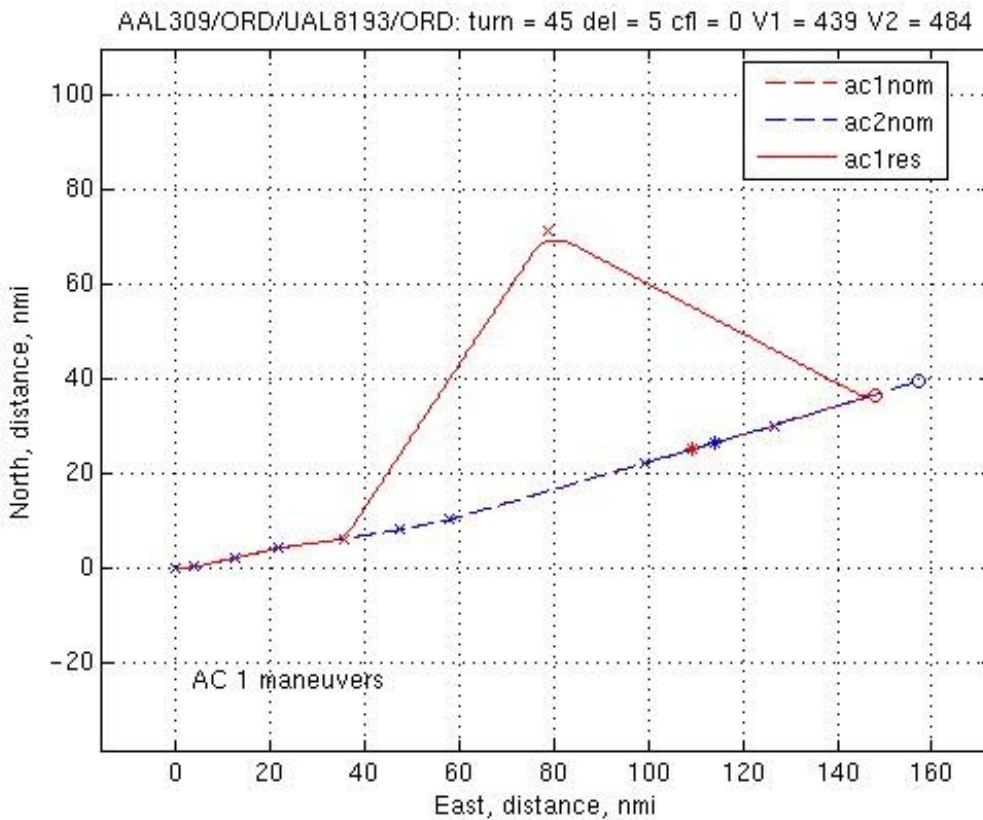


Figure 5.2. A path-stretch maneuver of 45° giving a delay of 5 min for AAL309.

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APPENDIX A

RHUMBLINE CALCULATIONS

A rhumbline is a line of constant heading that joins two points on a spherical surface. For typical distances between flight-plan waypoints, a rhumbline differs little from a great-circle route. This appendix reviews the equations for obtaining the rhumbline distance and heading given two waypoints, and for obtaining a waypoint given one waypoint and the distance and heading to it. First define the latitude and longitude of the waypoints as (φ_1, λ_1) and (φ_2, λ_2) , the rhumbline heading and distance between the waypoints as (X, L) , and R_E as the Earth radius.

Given the waypoint positions, the heading from waypoints 1 to 2 is given by

$$X = \text{atan2} [(\lambda_2 - \lambda_1), (d_2 - d_1)] \quad (\text{A.1})$$

where

$$d_1 = \log [\tan(\pi/4 + \varphi_1/2)]; \quad d_2 = \log [\tan(\pi/4 - \varphi_2/2)] \quad (\text{A.2})$$

The rhumbline distance is then

$$L = |R_E(\varphi_2 - \varphi_1) / \cos X| \quad (\text{A.3})$$

unless $\varphi_2 = \varphi_1$. In that case:

$$L = |R_E(\lambda_2 - \lambda_1) \cos \varphi_1| \quad (\text{A.4})$$

Solution of the inverse problem is used to find a turn-back waypoint given a heading and distance to it from a current point:

$$\varphi_2 = \varphi_1 + L \cos X / R_E, \text{ and } \lambda_2 = \lambda_1 + (d_2 - d_1) \tan X \quad (\text{A.5})$$

unless $\varphi_2 = \varphi_1$. In that case the longitude is

$$\lambda_2 = \lambda_1 + \text{sign}(X) L / (R_E \cos \varphi_1) \quad (\text{A.6})$$

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14. ABSTRACT An algorithm is described for a "path-stretch" maneuver that allows an aircraft to turn from its present course and turn back to a downstream waypoint while achieving a specified delay. The maneuver may be started while the aircraft is in climb, cruise, or descent, and will be independent of the vertical profile defined by its flight plan. This maneuver is particularly useful when two aircraft are "in trail" when the leading aircraft is slower, or when descending arrivals on converging routes are predicted to be in conflict near an arrival fix. While the specified delay may be achieved, there is no guarantee that the maneuver will clear a conflict. Therefore, it is essential that the process be part of a decision-support system that checks each "trial" maneuver for conflict before a clearance is issued to the aircraft.					
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