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An Algorithm for Level-Aircraft Conflict Resolution

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AN ALGORITHM FOR LEVEL-AIRCRAFT CONFLICT RESOLUTION

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I. INTRODUCTION

Much has been written about resolving level conflicts between two aircraft. Papers by Bilimoria and Paielli (refs. 1 and 2, respectively) at NASA review the literature on this topic, and present useful results. Yutaka and Erzberger (ref. 3) have compiled a comprehensive exposition of level-turn conflict resolutions. This monograph extends the work of Yutaka and Erzberger to allow a selection of turns (e.g., ±15°, ±30°, etc.) by one of the conflicting aircraft, while preserving the useful time and distance predictions provided by the turn algorithm.

This paper documents a simple and reliable level-turn algorithm included with a suite of automated resolutions for the Airspace Concept Evaluation System (ACES), an advanced air-traffic simulation (ref. 4). The suite is part of the Advanced Airspace Concept (AAC), described by Erzberger in a recent paper (ref. 5). This paper presents the turn algorithm along with a procedure for turning back to a waypoint to resume the original flight plan. The measure for comparing level turns is based on minimizing the delay required for an aircraft to complete its maneuver.

The paper begins with a review of the level conflict scenario, followed by conflict parameter definitions. Resolution of a level-altitude conflict with a single-aircraft turn is described, followed by a practical variant of the resolution algorithm that allows specified turns, an efficient return-to-flight plan procedure, an example, and finally, some concluding remarks. An appendix reviews the conditions for two solutions when the slower aircraft attempts the turn resolution.

II. CONFLICT SCENARIO

Consider two aircraft, A and B, flying at the same flight level in a uniform wind field. The airspeeds and headings are assumed to be constant and known. The aircraft trajectories can be estimated from the flight-plan waypoints and verified by real-time tracking of aircraft position. If estimates of the wind field along each flightpath are available, then ground speeds may be calculated. It is assumed that an air-traffic system is performing periodic conflict detection, using tracking data, aircraft performance information, and flight-plan trajectory predictions.

A typical conflict-detection scenario is shown in figure 2.1. The line AB is the line-of-sight (LoS) vector s_0 between the aircraft, and v_A and v_B are the aircraft (ground) velocity vectors. The circle

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centered at aircraft B has radius R_M, which is the allowable minimum separation (5 nmi outside terminal airspace) to avoid conflict. In a frame moving with aircraft B, aircraft A proceeds along the relative velocity vector v_R, which is defined as

$$v_R = v_A - v_B \tag{2.1}$$

To avoid conflict, the relative velocity vector must be directed along or outside the dashed lines tangent to the circle (conflict zone). If the allowable minimum separation is R_M, then the required angle between v_R and s_0 is

$$\beta = \pm \sin^{-1}(R_M/S_0) \tag{2.2}$$

where S_0 is the length of the LoS vector. Note that when $S_0 \le R_M$, the aircraft are already in violation. It is clear that a potential conflict must be detected while $|\beta| < 90°$ in order to effect a resolution. (Angles shown as increasing clockwise (cw) are positive.)

The situation shown in figure 2.1 indicates that a conflict exists, that is, the dashed line containing the relative velocity vector v_R penetrates the conflict circle of radius R_M (at point F). Horizontal resolution can be achieved if one or both aircraft maneuver to cause v_R to rotate about point A by an angle $\mu = \beta - \alpha$, so that the relative velocity vector lies along either tangent line. A rotation of the velocity triangle ACP to either line will resolve the conflict; this rotation can always be accomplished by *each* aircraft simultaneously turning by the angle μ. For operational reasons, however, single-aircraft maneuvers are preferred and are considered here.

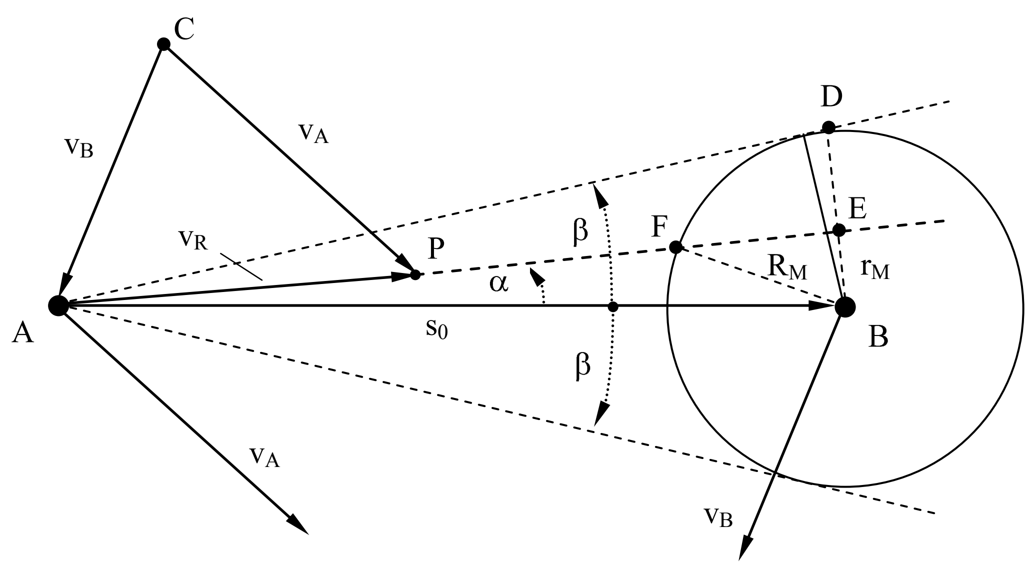

Figure 2.1. Position and velocity geometry for a level conflict scenario.

Clearly, no aircraft can turn instantaneously. For conflicts detected early enough (at least four minutes before minimum separation), turn dynamics should not be a concern. The addition of constant-radius turns is straightforward, but is not covered here. A turn in a significant wind field will of course affect the ground speed of the turning aircraft, but winds are considered negligible in what follows.

III. CONFLICT PARAMETERS

To describe the conflict scenario of figure 2.1 quantitatively, the magnitude and heading of vector v_R must be computed. Note that heading angles are measured positive cw from the vertical (North) on the page. To compute the relative velocity vector, first assign heading angles ψ_A, ψ_B and speeds V_A, V_B to the velocity vectors v_A and v_B, respectively. To obtain the speed V_R and heading ψ_R of the relative velocity vector, apply the Law of Sines to the velocity triangle ACP. The result is given by

$$V_A / \sin(\psi_B - \psi_R) = V_B / \sin(\psi_A - \psi_R) \quad (3.1)$$

Use trigonometric identities to obtain

$$\psi_R = \tan^{-1}(N / D);\ V_R = (N^2 + D^2)^{1/2}$$

where (3.2)

$$N = V_A \sin\psi_A - V_B \sin\psi_B;\ D = V_A \cos\psi_A - V_B \cos\psi_B.$$

If the aircraft do not maneuver to avoid the conflict, their minimum separation will occur at point E (the line segments AE and DB are perpendicular). Minimum separation is represented by the segment BE. Its length and the predicted time to reach point E are given by

$$r_M = S_0 \sin|\alpha|;\ t_M = S_0 \cos\alpha / V_R \quad (3.3)$$

where S_0 is the initial separation at the time of conflict prediction and V_R is the relative speed.

An important parameter for representing a conflict is the predicted time to reach first loss of separation (point F in figure 2.1). The distance to first loss along the relative velocity vector is the difference AE – FE, which is equivalent to

$$d_1 = S_0 \cos\alpha - (R_M^2 - r_M^2)^{1/2} \quad (3.4)$$

Recall that S_0 is the LoS distance between aircraft A and B at the conflict-detection instant time t_0. Hence, the predicted time to reach first loss of separation is

$$t_1 = d_1 / V_R \quad (3.5)$$

A parameter for specifying a resolution is the time either (or both) aircraft must fly after turning to reach a suitable turn-back point (shown as point D in fig. 2.1). At this point, the maneuvering aircraft may turn back toward its original track and proceed to a downstream waypoint. The turn-back point is defined so that a heading change of the relative velocity vector of no more than -2μ will avoid reentering the conflict zone. Since the line segments AE and DB are perpendicular, it can be shown that the distance between points A and D and the time to reach turnback are given by

$$d_T = S_0(\cos\beta + \sin\beta \tan\mu) = S_0 \cos\alpha / \cos\mu;\ t_T = d_T / V_R^* \tag{3.6}$$

where V_R^* is the speed of the resolved relative velocity vector along AD.

If the conflicting aircraft are indeed at the same flight level in cruise, the time t_T in eq. (3.6) is useful for computing the turn-back point. For example, if aircraft A turns to avoid the conflict, it should turn back from its vector leg after a distance

$$d_V = V_A t_T \tag{3.7}$$

The next section describes an algorithm for calculating a turn required for a single aircraft A to clear a conflict.

IV. SINGLE AIRCRAFT RESOLUTION

Resolution schemes that have *one* aircraft making a heading change have been devised (ref. 1-3). These schemes may be operationally preferable to providing simultaneous advisories to both aircraft in conflict. Refer to the expanded velocity diagram of figure 4.1, where the information shown in figure 2.1 has been simplified. This diagram makes it possible to easily visualize a resolution performed by aircraft A, either by turning counter clockwise (ccw) (vector v_A rotates about point C) until the relative velocity vector lines up with the upper dashed tangent line at point c, or turning cw until the relative velocity vector lines up with the lower dashed line at point e. Notice that in this example, A is the faster aircraft; hence, there is one valid solution for *each* tangent line.

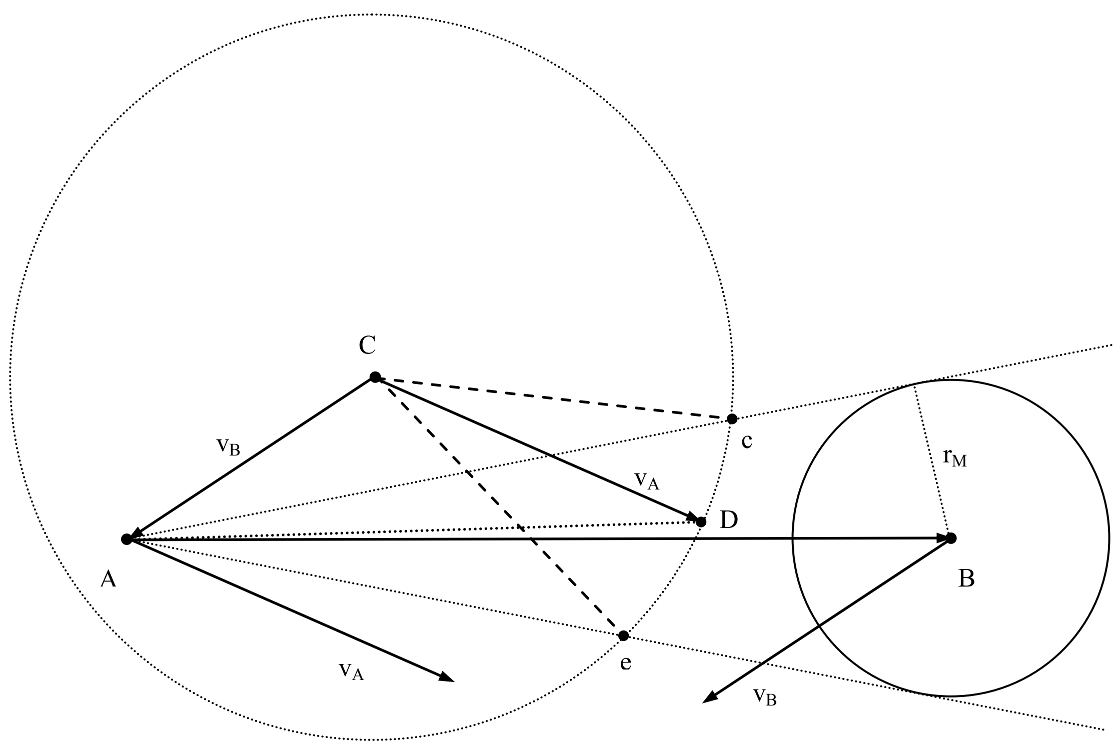

Figure 4.1. Heading changes for aircraft A to resolve a conflict ($V_A > V_B$).

The resolved heading for the faster aircraft is obtained by applying the Law of Sines to the new velocity triangle (either ACc or ACe). The result is

$$\psi^*_A = \psi^*_R + \sin^{-1}[\sigma_V \sin(\psi_B - \psi^*_R)]; \;\; \sigma_V = V_B / V_A \tag{4.1}$$

where ψ^*_A is the new heading of aircraft A (either Cc or Ce), and ψ^*_R is the heading of the resolved relative velocity vector (either segment Ac or Ae). The circle is chosen to realize a specified minimum separation (e.g., $r_M = 7$ nmi).

For an example of this resolution maneuver, suppose that

$V_A = 356.0$ kn, $\psi_A = 133°$, $V_B = 300.0$ kn, $\psi_B = -157°$

From eq. (3.2) the magnitude and heading of the relative velocity vector are

$V_R = 379.1$ kn; $\psi_R = 85°$

Since the initial positions of aircraft A and B are known, the magnitude and heading for the LoS vector between them can be computed. Suppose that the LoS vector has length $S_0 = 22$ nmi and heading $\psi_0 = 90°$, so that $\alpha = -5°$. The predicted time to first loss (penetration of the $R_M = 5$ circle), calculated using eqs. (3.5) and (3.6), is $t_1 = 2.7$ min. Solution of eq. (2.2) to achieve a desired minimum separation of $r_M = 7$ nm yields a required $\beta = \pm 18.6°$.

Conflict resolution will be achieved by turning aircraft A: Solve eq. (4.1) for either ccw or cw rotation of the vector v_A (in a negligible wind field) to obtain

for $\mu = -13.5°$: $\psi^*_A = 110.5°$; $\psi^*_R = 71.4°$; $V^*_R = 475.3$ kn; $d_V = 16.9$ nmi (left turn)

for $\mu = 23.6°$: $\psi^*_A = 165.7°$; $\psi^*_R = 108.6°$; $V^*_R = 216.3$ kn; $d_V = 39.4$ nmi (right turn)

Hence, aircraft A must turn left (behind aircraft B) by 22.5° or right (in front of B) by 32.7° to resolve the conflict. A "turn-in-front" usually requires a greater distance before the aircraft can turn back to resume its flight-plan route.

For resolution by the slower aircraft, refer to figure 4.2, and again rotate vector v_A about point C until the relative velocity vector lines up with either tangent line. Here it is seen that a v_A rotation yields two intersections with the lower tangent line (at e_1 and e_2), and two intersections with the upper tangent line (at c_1 and c_2). In this case, there are two valid solutions for the slower aircraft with *each* tangent line. In some cases, however, there may be no solutions for one of the tangent lines (e.g., shorten the length of v_A in fig. 4.2). The limiting conditions are derived in appendix A.

When two valid solutions exist for the slower aircraft along a given tangent line, the first is given by eq. (4.1); the second solution is

$$\psi^*_A = \psi^*_R - \sin^{-1}[\sigma_V \sin(\psi_B - \psi^*_R)] + 180° \tag{4.2}$$

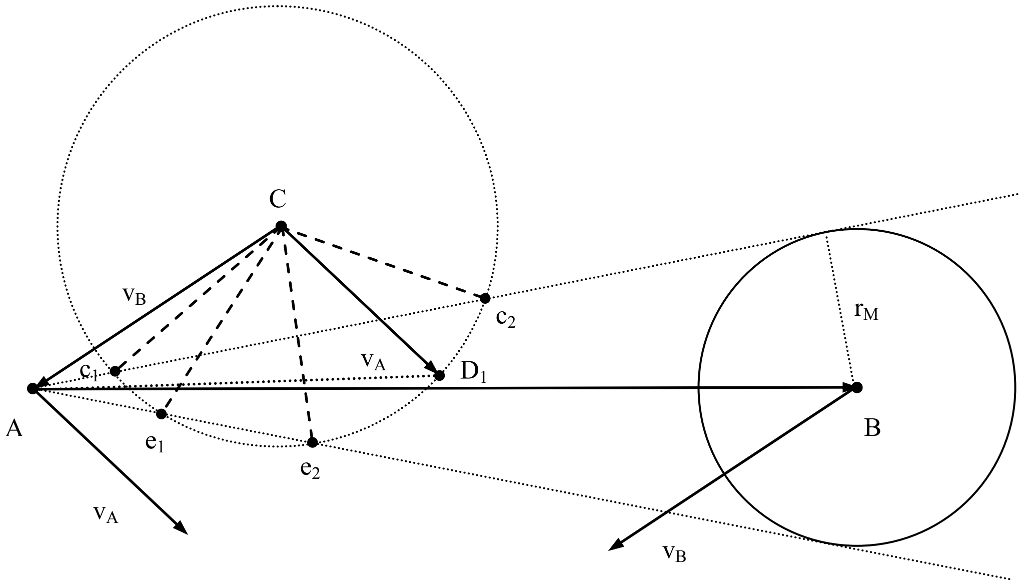

Figure 4.2. Heading changes for aircraft A to resolve a conflict ($V_A < V_B$).

Interchange the aircraft labels of the previous example so that A again is the turning aircraft. Now the speeds and headings are

$V_A = 300.0$ kn, $\psi_A = -157°$, $V_B = 356.0$ kn, $\psi_B = 133°$

The speed and heading of the relative velocity vector are

$V_R = 379.1$ kn; $\psi_R = -95°$

Recall that the LoS vector has magnitude $S_0 = 22$ nm; its heading is now $\psi_0 = -90°$ ($\alpha = -5°$). The predicted time to first loss is still 2.7 min, and $r_M = 7$ nm again requires that $\beta = \pm\ 18.6°$.

Conflict resolution will again be achieved by turning aircraft A. However, each of the two velocity-vector rotations must be checked for validity; i.e., the ccw rotation is $\mu = -13.5°$, while the cw rotation is $\mu = 23.6°$. From eq. (A.3), the valid range is $-9.4° < \mu < 105.5°$. Hence the ccw rotation yields *no* solution, while there will be two solutions for the cw rotation. For $\mu = 23.6°$, the solutions obtained from eqs. (4.1) and (4.2) (assuming a negligible wind field) are

$\psi_A^* = -100.9°$; $\psi_R^* = -71.4°$; $V_R^* = 585.4$ kn; $d_V = 12.3$ nmi (right turn)

$\psi_A^* = 138.0°$; $\psi_R^* = -71.4°$; $V_R^* = 62.8$ kn; $d_V = 114.3$ nmi (left turn)

Hence, aircraft A must turn right (behind aircraft B) by 56.1° or left (in front of B) by 65° to resolve the conflict. Note that the turn in front requires a much longer vector leg.

V. A PRACTICAL TURN ALGORITHM

In the previous section, the turn was dependent on the specified minimum separation. That is, the value of β from eq. (2.2) defined a heading for the relative velocity vector ψ_R^*; then the heading ψ_A^* for the maneuvering aircraft was found using eq. (4.1) (or eqs. (4.1) and (4.2)). However, in a significant wind field, the ground speed will change along the new path, requiring either an airspeed change to maintain constant ground speed, or iteration to maintain constant airspeed. Neither alternative is explored here.

A more practical approach is to first choose a set of turns for the maneuvering aircraft (e.g., ±15°, ±30°, etc.); for each turn calculate the relative velocity vector, and check to see if some desired minimum separation will be met. This approach is consistent with current operational practice. Furthermore, in the presence of a significant wind field, the ground speed along each of the proposed new headings can be estimated, using the (unchanged) airspeed and wind estimates.

To illustrate for aircraft A, a desired turn $\Delta\psi$ will yield a target heading

$$\psi_A^* = \psi_A + \Delta\psi \tag{5.1}$$

Next, compute the new ground-speed V_A^* and use eq. (3.1) to calculate the resolved relative velocity vector (ψ_R^*, V_R^*). The required rotation from the LoS vector and the minimum separation achieved with this turn are then

$$\beta = \psi_R^* - \psi_0;\ r_M = S_0 \sin|\beta| \tag{5.2}$$

Now, if $|\beta| < 90°$ and if r_M is greater than some desired minimum separation (say 7 nmi), continue with this turn maneuver by calculating t_T, the predicted time to the turn-back point from eq. (3.6) and d_V, the distance along the vector leg to turn back from eq. (3.7). Note that if A is the slower aircraft, the value of $\mu = \beta - \alpha$ obtained in this approach will always satisfy the inequality of eq. (A.4). Finally, check to ensure that $\psi_R^* = \psi_R + \mu$.

If the predicted r_M is smaller than the desired minimum separation, abandon the turn trial and try a turn in the opposite direction (or go to the next larger turn, if possible). Note that if $|\beta| \geq 90°$ the algorithm is not valid. However, here a simple vector turn still may result in a resolution of the conflict. In this case, choose the turn-back distance to be $d_V = 2d_1$, where d_1 is the distance to first loss for aircraft A.

Now we return to the first example of section IV, and consider a turn for aircraft A that yields a minimum separation of at least 7 nmi. For a left turn of 25°, the target heading will be $\psi_A^* = 133 - 25 = 108°$. The resulting velocity vector, minimum separation, and turn-back distance is

$$\psi_R^* = 70°;\ V_R^* = 485.1 \text{ kn};\ r_M = 7.5 \text{ nmi};\ d_V = 16.6 \text{ nmi}$$

Note that the velocity vector has, in fact, been rotated ccw by $\mu = 15°$ to provide a resolution with predicted minimum separation of 7.5 nmi.

The results of the second example of section IV imply that selection of a cw turn from the set (15°, 30°, 45°, 60°) would be limited to 60°. The other right turns would yield predicted minimum separations less than 7 nmi. For a right turn of 60°, the target heading is $\psi^*_A = -97°$, and the resulting velocity vector, minimum separation, and turn-back distance are

$$\psi^*_R = -69.7°;\ V^*_R = 595 \text{ kn};\ r_M = 7.6 \text{ nmi};\ d_V = 12.2 \text{ nmi}$$

VI. MANEUVER COMPLETION

To complete a turn maneuver, the aircraft returns to its original flightpath, preferably to a designated flight-plan waypoint. A candidate return waypoint must be within suitable limits for the range (LoS distance) from the initial point to the waypoint. The minimum range is set to twice the distance to first loss, a distance that is usually available in the conflict data record; it can also be estimated using eq. (3.4). The maximum range is set to the lesser of the range to the final fix and 500 nmi. The final fix is generally excluded from return candidacy. If no flight-plan waypoint exists within the range window, then one or more waypoints may be inserted and the one ultimately selected added to the flight-plan set. A typical level conflict resolution maneuver for a cw turn is shown in figure 6.1.

A candidate return waypoint is also tested to satisfy the conditions

$$d_R > d_V \text{ and } |\varphi_T| < 90° \tag{6.1}$$

where d_R is the distance to the return waypoint from the turn-back point, d_V is the distance from the initial point to turn back, and φ_T is the turn angle between the vector leg and the return leg of the maneuver. This test is based on operational considerations. Note that if the test fails for the *last* waypoint candidate, the value of d_V can be reduced until the test is satisfied.

Following completion, a turn maneuver should be checked for conflicts with other aircraft in the airspace. If several candidate turns for resolving a conflict exist, they can be ranked using a delay metric. This metric is defined as the difference in the estimated times to the return waypoint along the resolution path and the original path. In most cases maneuvers would be evaluated in the order of increasing delay.

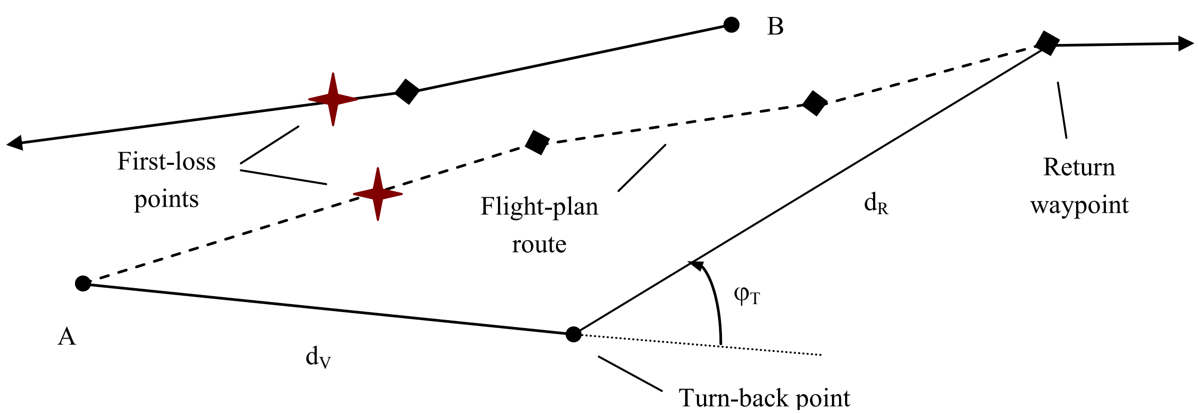

Figure 6.1. Typical level conflict-resolution plan view.

VII. A CASE STUDY

The level-turn algorithm described in sections V and VI was developed and tested in an environment with MATLAB™ software from The MathWorks, Inc.; it was then converted to Java, and implemented with the AAC auto-resolution software in the ACES air-traffic simulation (ref. 5). For the tests being conducted at Ames Research Center, the simulation uses flight-plan data from one day of flights in the Cleveland airspace. The aircraft start from airports in the United States at scheduled departure times, and fly according to their filed flight plans. Each aircraft within the Cleveland Center is checked for conflict with all other aircraft in the Center every two minutes, and a conflict list is sent to the AAC auto-resolution module. The ACES–AAC interface is shown in figure 7.1.

This section provides a case study of one level conflict pair for which several trial plans are created. The trial plans consist of level-turn maneuvers for either aircraft, which would be created in the AAC module, and ordered so that the maneuver with the least delay from the original route would be sent to ACES to be checked for feasibility and conflicts. This study, however, was performed with MATLAB using the level resolution algorithms described in the previous sections, applied to ACES conflict data. The same algorithms, with some practical constraints, have been implemented in the AAC module.

The case chosen for this paper is a conflict between flights AAL309 (an MD-80) and UAL8193 (a B757), flying at 31,000 ft through the Cleveland Center, both en route to Chicago. The data record accompanying each conflict detected by ACES and sent to the AAC module includes, for each aircraft, position, velocity, and time at the initial point, the first-loss point, the minimum separation point, and each flight-plan waypoint. Data for the initial point should be considered "measured," the rest, "predicted" by the ACES trajectory generator. A summary of the conflict data is given in table 7.1.

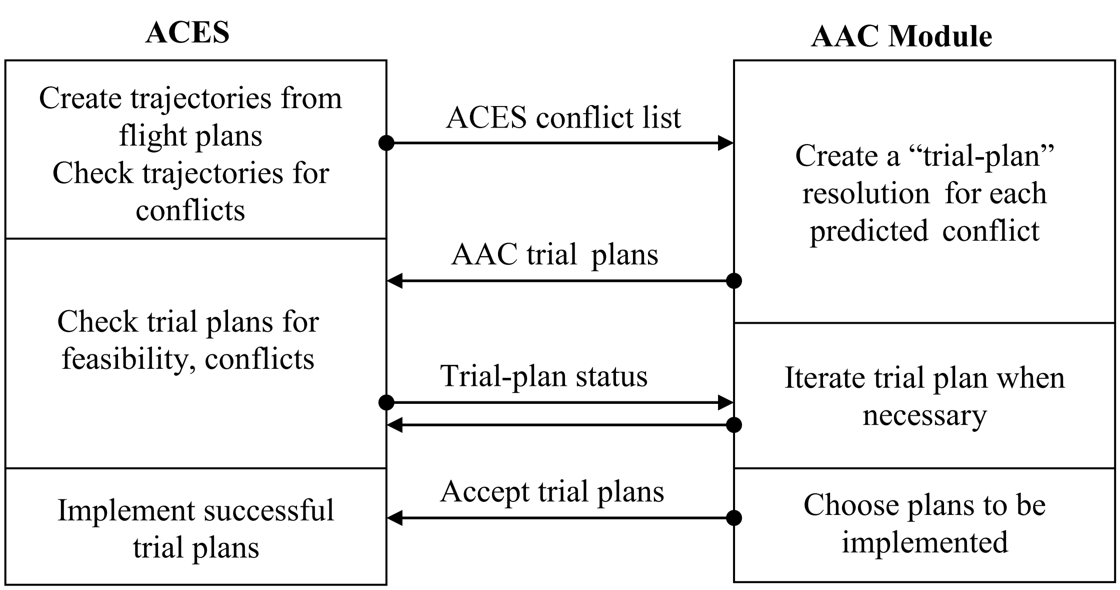

Figure 7.1. Interface of ACES and the AAC auto-resolution module.

TABLE 7.1. CONFLICT DATA FOR LEVEL-CONFLICT CASE STUDY.

	Initial T_0 = 0			First-loss T_1 = 5.5 min			Min Separation T_M = 6.3 min		
	Alt, ft	Spd, kn	Hdg, deg	Alt, ft	Spd, kn	Hdg, deg	Alt, ft	Spd, kn	Hdg, deg
AAL	31,000	438.8	–107.0	31,000	439.0	–106.7	31,000	439.0	–105.7
UAL	31,000	483.5	–106.9	31,000	485.0	–106.9	31,000	485.0	–106.9

Calculations of LoS and relative velocity vectors, alpha, and prediction of minimum separation (from section III) yield

$$S_0 = 9.1 \text{ nmi}, \psi_0 = 72.4°;\ V_R = 44.7 \text{ kn}, \psi_R = 71.0°;\ \alpha = -1.4°;\ r_M = 0.2 \text{ nmi}$$

Examination of the data reveals that the aircraft have all flight-plan waypoints in common; i.e., they are "in trail," only 9.1 nmi apart at the initial point. The speeds differ by about 45 kn, and the UAL flight is predicted to overtake the AAL flight in 6.3 min (r_M = 0.2 nmi) if no action is taken. Trial turn resolutions of ±15° and ±30° will be attempted for each aircraft. A turn of AAL will allow the faster UAL to be ahead when it passes the AAL return waypoint; however, a UAL turn may *not* allow the faster aircraft to be ahead when it returns to its route. Turning the slower AAL aircraft would probably be the first choice of an ATC controller. The trial-turn results are shown in table 7.2.

It should be noted that the MATLAB resolution software includes a level conflict check. The inter-waypoint paths are tested for separation every 5 sec for each trial plan, with a look-ahead time of 12 min. Although no turn dynamics are modeled, this check is useful for monitoring resolution performance. In the column labeled "r_M (test)" it is seen that all turns appear to meet the requirement that $r_M \geq 7$ nmi. For the ±30° turns of the faster UAL aircraft, however, the calculation of eq. (5.2) requires that $|\beta| > 90°$. Hence the algorithm predictions are no longer valid. For both these turns, however, the vector maneuver is conflict-free, at least for the first 12 min. For all other turns, the minimum separation predicted by the algorithm compares closely with the test.

TABLE 7.2. SUMMARY OF TURN MANEUVERS ($R_M \geq 7$ NMI).

a/c	Turn	r_M (pred)	r_M (test)	d_V, nmi	φ_1	d_R, nmi	Delay, min	wpt
AAL	–15°	8.1	8.0	72.0	34.8°	74.3	0.9	7
	+15°	8.0	8.0	63.0	–23.8°	69.7	0.3	6
	–30°	8.3	8.3	41.8	50.8°	61.7	1.4	3
	+30°	8.2	8.2	36.1	–45.4°	64.6	1.0	3
UAL	–15°	8.9	8.8	78.4	33.3°	86.4	0.9	8
	+15°	8.9	8.9	76.6	–24.1°	84.0	0.4	8
	–30°	*	9.1	89.0	62.4°	96.1	3.4	8
	+30°	*	9.1	89.0	–53.2°	90.7	2.2	8

* $|\beta| > 90°$: algorithm not valid (vector turn used).

The turn of 15° for the AAL aircraft, shown in the plan view of figure 7.2, allows the faster UAL aircraft to pass in front and results in the least delay (0.3 min). In the ACES–AAC implementation, this maneuver would be tried first. The next smallest delay (0.4 min) is for the turn of 15° for the UAL aircraft. While the vector leg of this maneuver is conflict-free, "stretching" the path of the faster aircraft may cause a problem near the return waypoint if both aircraft are still at cruise altitude. The largest delay (3.4 min) is required for the 30° vector turn of the faster UAL aircraft; again the aircraft will likely be in conflict near the return waypoint. Observe that both UAL vector turns require a longer path before returning to the flight plan (at the last candidate return waypoint).

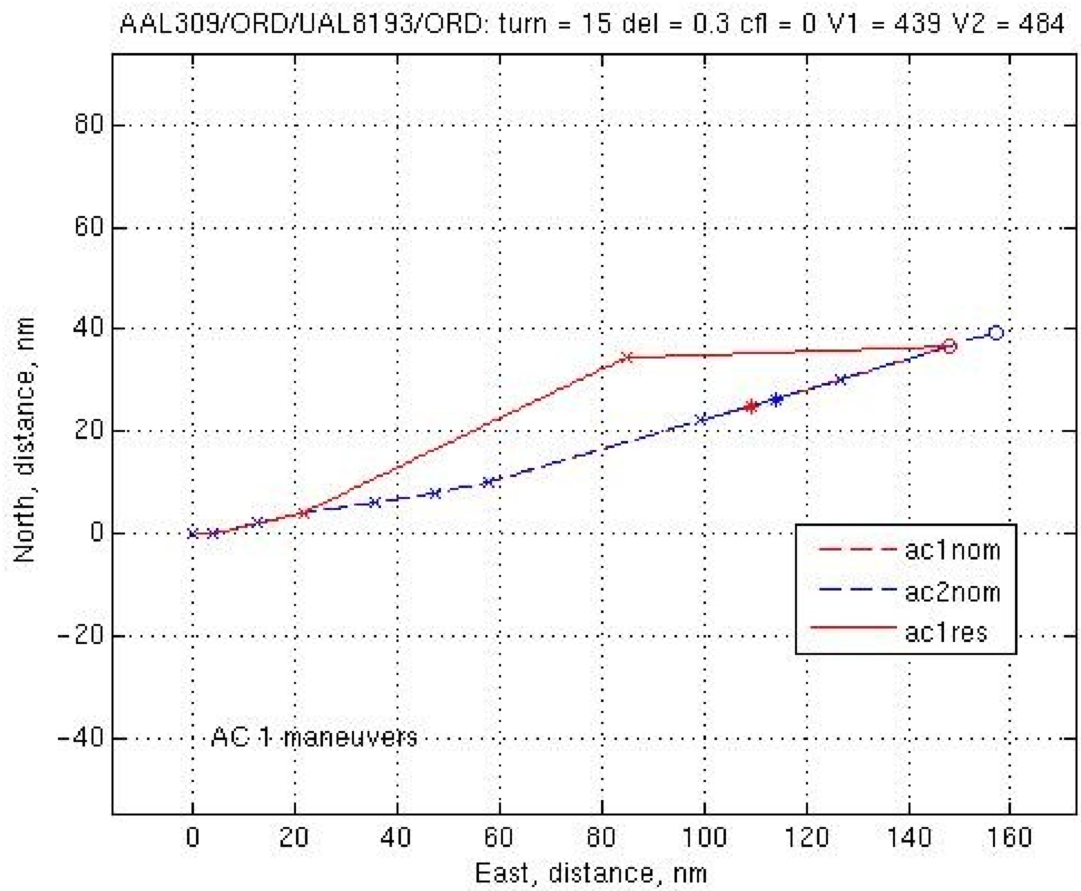

Figure 7.2. A cw turn resolution of 15° for the slower AAL aircraft.

VIII. CONCLUDING REMARKS

This paper has documented a simple and reliable level-turn resolution algorithm that has been included with a suite of automated resolutions in an advanced air-traffic simulation. The long-term goal of this work is to integrate the Advanced Airspace Concept with the real-time Center-TRACON Automation System (ref. 6). This paper outlines the level-turn algorithm and includes a procedure to turn back to a waypoint and resume the original flight plan. The measure for comparing level turns is based on minimizing the delay required for an aircraft to complete its maneuver.

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APPENDIX A

This appendix reviews the conditions for a single-aircraft heading resolution to resolve a horizontal conflict when aircraft A maneuvers and its speed V_A is less than the speed of B (V_B). In this case, it may be possible to rotate v_A to obtain two intersections for a given rotation μ of the relative velocity vector; otherwise there will be no intersections. Figure A.1 shows the limit for two solutions to occur: a rotation of the relative velocity vector from heading ψ_R by an angle μ_{max} (cw) to a resolution heading $\psi^*_{R\,max}$, and by an angle μ_{min} (ccw) to a heading $\psi^*_{R\,min}$.

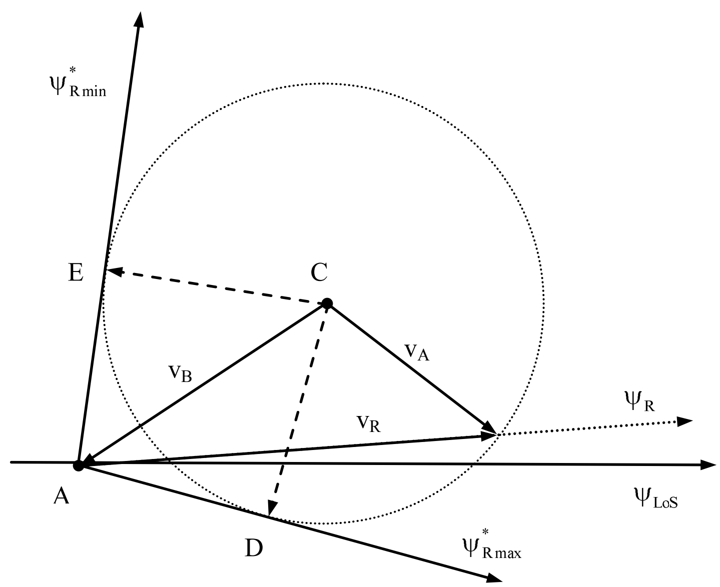

Figure A.1. The limiting case for two solutions with $V_A < V_B$ (A maneuvers).

Note that both resolution vectors are tangent to a circle of radius V_A and that the included angles of the *right* triangles CAE and CAD are equal, of value

$$\theta = \sin^{-1}(V_A / V_B) \tag{A.1}$$

Simple trigonometry applied to the rotations shown in figure A.1 yields

$$[(\psi_R + \mu_{min}) - \psi_B] + \theta = 180° \text{ and } [\psi_B - (\psi_R + \mu_{max})] + \theta = 180° \tag{A.2}$$

which leads to the limit values for μ

$$\mu_{min} = \psi_B - \psi_R + 180° - \sin^{-1}(V_A / V_B) \text{ and } \mu_{max} = \psi_B - \psi_R - 180° + \sin^{-1}(V_A / V_B) \tag{A.3}$$

Hence, for two heading resolutions by the slower aircraft to exist for a rotation of the velocity vector by an angle μ,

$$\mu_{min} < \mu < \mu_{max} \tag{A.4}$$

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14. ABSTRACT
A turn algorithm is described for resolving a conflict between two aircraft flying with constant velocities at the same altitude. The procedure yields the turn angle for one aircraft to achieve the required minimum-separation distance between the conflicting aircraft, as well as the point to return to its flight-plan route. A practical variant of the algorithm is described that permits a specified turn with an efficient return-to-flight plan procedure. This algorithm is part of a suite of automated resolutions in an advanced air-traffic simulation.

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